Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/102-4.3.1.3-d-sin-^m-a+b-tan-^n

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September 27, 2022 Compiled on September 27, 2022 at 1:42pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (91)	0.00 (0)
Mathematica	98.90 (90)	1.10 (1)
Fricas	91.21 (83)	8.79 (8)
Maple	91.21 (83)	8.79 (8)
Mupad	91.21 (83)	8.79 (8)
Giac	90.11 (82)	9.89 (9)
Maxima	86.81 (79)	13.19 (12)
Sympy	8.79 (8)	91.21 (83)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

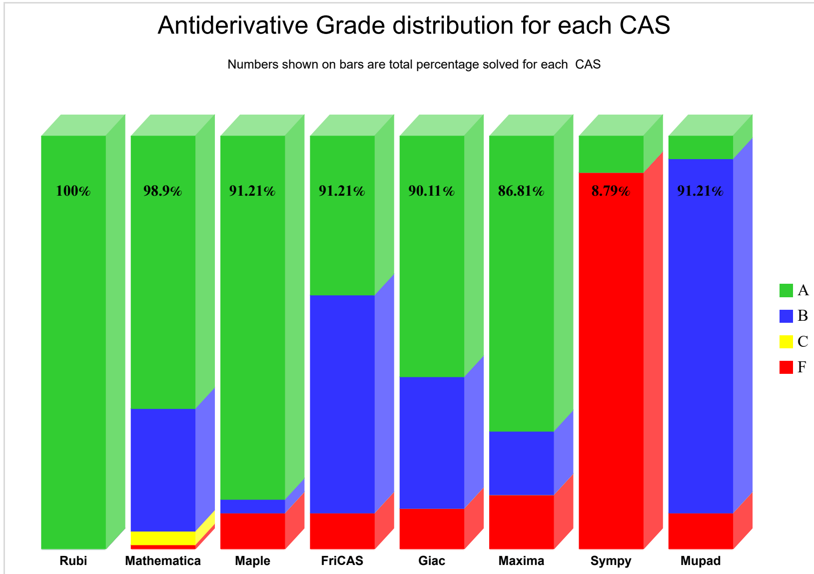
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

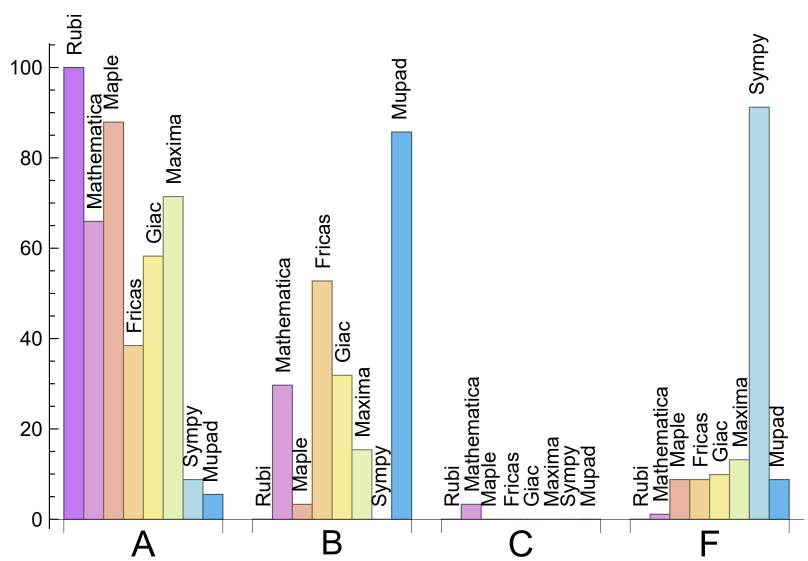
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	87.91	3.30	0.00	8.79
Maxima	71.43	15.38	0.00	13.19
Mathematica	65.93	29.67	3.30	1.10
Giac	58.24	31.87	0.00	9.89
Fricas	38.46	52.75	0.00	8.79
Sympy	8.79	0.00	0.00	91.21
Mupad	N/A	85.71	0.00	8.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	8	100.00 %	0.00 %	0.00 %
Fricas	8	100.00 %	0.00 %	0.00 %
Giac	9	88.89 %	11.11 %	0.00 %
Maxima	12	66.67 %	0.00 %	33.33 %
Sympy	83	83.13 %	7.23 %	9.64 %
Mupad	8	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

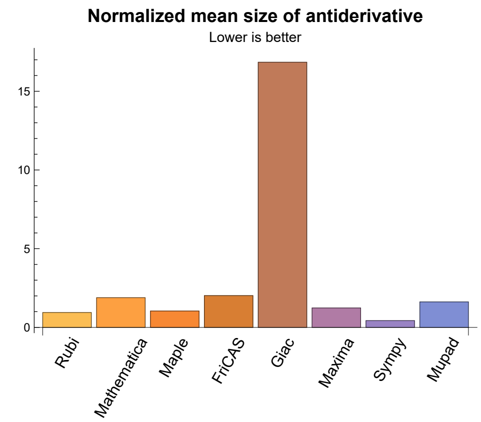
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

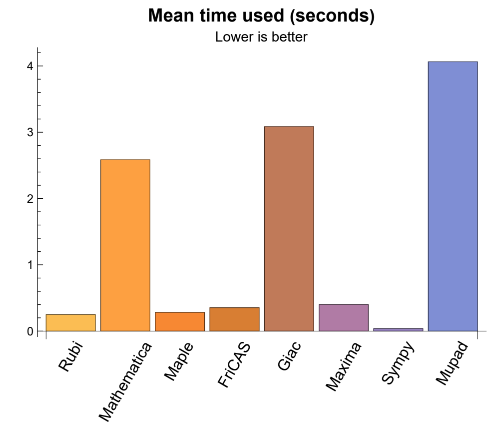
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	136.40	0.95	109.00	1.00
Mathematica	2.58	278.01	1.89	162.00	1.64
Maple	0.28	131.59	1.04	106.00	0.98
Maxima	0.40	185.22	1.24	120.00	1.08
Fricas	0.35	285.42	2.02	176.00	1.97
Sympy	0.04	17.00	0.43	8.50	0.31
Giac	3.08	2374.78	16.85	191.00	1.63
Mupad	4.06	234.31	1.62	146.00	1.25

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{83, 88, 89, 90, 91\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {48}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

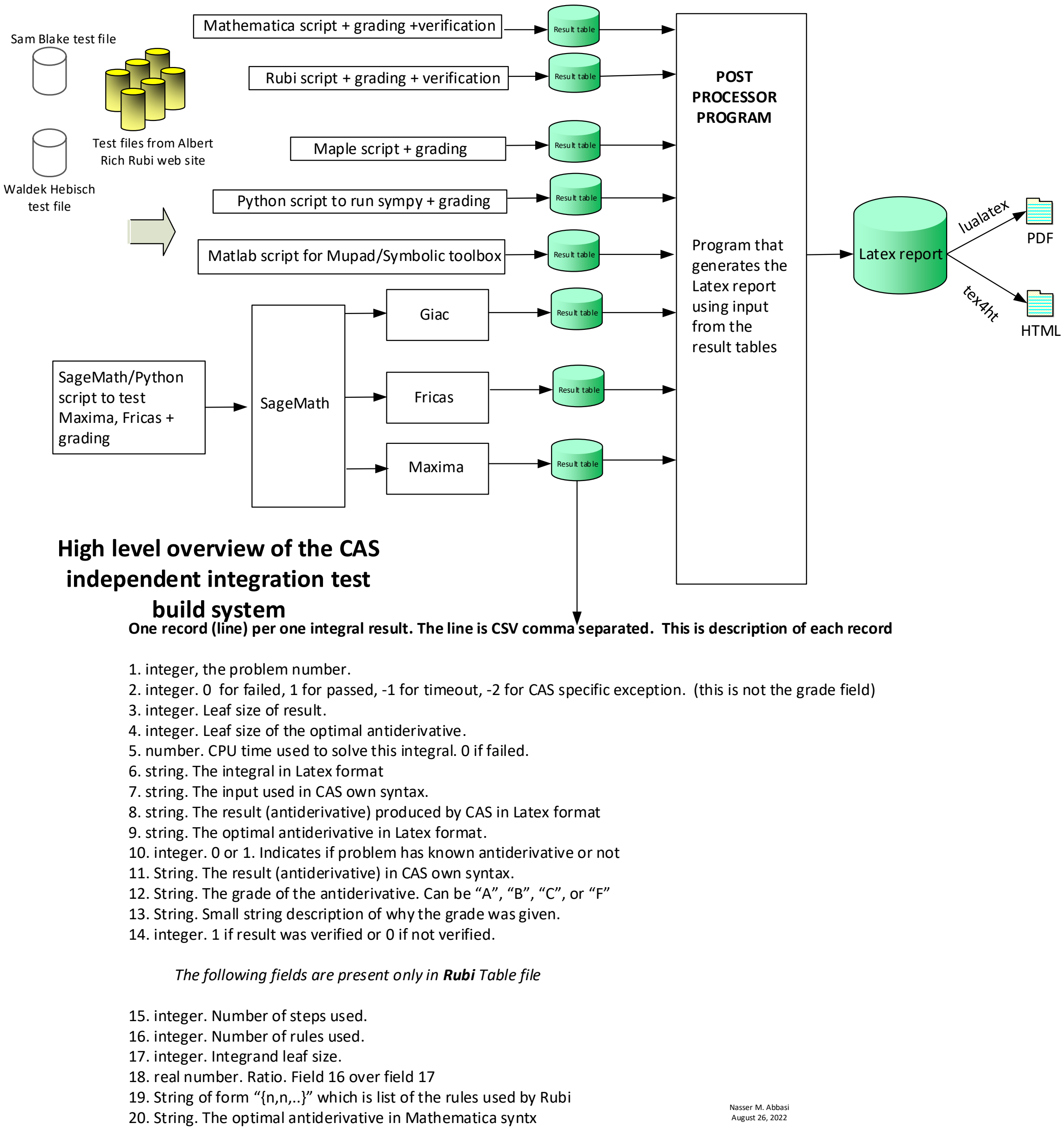
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91 }

B grade: { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 61, 66, 70, 71, 75, 76, 77, 84 }

C grade: { 18, 20, 78 }

F grade: { 82 }

2.1.3 Maple

A grade: { 1, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 88, 89, 90, 91 }

B grade: { 2, 4, 7 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.4 Maxima

A grade: { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 9, 51, 53, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade: { }

F grade: { 1, 2, 3, 4, 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 57, 59, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 47, 48, 49, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 83, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 83, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 9, 11, 12, 13, 14, 15, 18, 22, 23, 24, 25, 32, 33, 34, 37, 42, 43, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 41, 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.8 Mupad

A grade: { 83, 88, 89, 90, 91 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	66	0	39	51	53	49
N.S.	1	1.00	0.86	0.85	0.00	0.50	0.65	0.68	0.63
time (sec)	N/A	0.049	0.098	0.109	0.000	0.317	0.083	0.440	3.708

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	81	0	26	37	71	59
N.S.	1	1.00	1.76	2.79	0.00	0.90	1.28	2.45	2.03
time (sec)	N/A	0.099	0.024	0.096	0.000	0.316	0.099	0.427	3.865

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	47	0	25	31	41	35
N.S.	1	1.00	0.78	0.94	0.00	0.50	0.62	0.82	0.70
time (sec)	N/A	0.038	0.107	0.111	0.000	0.323	0.066	0.417	3.745

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	47	0	14	17	33	39
N.S.	1	1.00	1.74	2.47	0.00	0.74	0.89	1.74	2.05
time (sec)	N/A	0.062	0.016	0.118	0.000	0.319	0.053	0.415	3.828

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	31	21	28	25	0	22	20
N.S.	1	1.00	1.94	1.31	1.75	1.56	0.00	1.38	1.25
time (sec)	N/A	0.061	0.022	0.099	0.473	0.328	0.000	0.426	3.800

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	15	20	17	25	0	18	19
N.S.	1	1.00	0.83	1.11	0.94	1.39	0.00	1.00	1.06
time (sec)	N/A	0.024	0.023	0.134	0.337	0.318	0.000	0.406	3.738

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	75	42	59	73	0	46	41
N.S.	1	1.00	3.12	1.75	2.46	3.04	0.00	1.92	1.71
time (sec)	N/A	0.092	0.025	0.161	0.404	0.332	0.000	0.428	3.729

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	12	36	0	12	13
N.S.	1	1.00	1.53	0.79	0.63	1.89	0.00	0.63	0.68
time (sec)	N/A	0.028	0.023	0.163	0.415	0.311	0.000	0.435	3.624

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	83	123	0	62	57
N.S.	1	1.00	3.48	1.45	2.08	3.08	0.00	1.55	1.42
time (sec)	N/A	0.106	0.028	0.126	0.391	0.321	0.000	0.434	3.696

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	24	54	0	24	27
N.S.	1	1.00	1.11	0.76	0.65	1.46	0.00	0.65	0.73
time (sec)	N/A	0.033	0.023	0.111	0.389	0.312	0.000	0.436	3.601

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	103	80	91	97	0	12356	121
N.S.	1	1.00	1.02	0.79	0.90	0.96	0.00	122.34	1.20
time (sec)	N/A	0.055	0.052	0.187	0.431	0.362	0.000	1.655	3.804

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	73	87	74	0	1066	155
N.S.	1	1.00	0.99	0.88	1.05	0.89	0.00	12.84	1.87
time (sec)	N/A	0.109	0.091	0.161	0.684	0.345	0.000	0.591	3.787

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	60	70	74	0	5350	87
N.S.	1	1.00	1.03	0.87	1.01	1.07	0.00	77.54	1.26
time (sec)	N/A	0.048	0.034	0.150	0.363	0.346	0.000	0.998	3.828

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	52	52	47	0	413	50
N.S.	1	1.00	1.14	1.06	1.06	0.96	0.00	8.43	1.02
time (sec)	N/A	0.063	0.059	0.135	0.552	0.349	0.000	0.498	3.815

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	40	46	49	0	1236	53
N.S.	1	1.00	1.30	1.08	1.24	1.32	0.00	33.41	1.43
time (sec)	N/A	0.026	0.031	0.134	0.330	0.341	0.000	0.528	3.820

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	52	40	46	58	0	49	86
N.S.	1	1.00	2.00	1.54	1.77	2.23	0.00	1.88	3.31
time (sec)	N/A	0.021	0.024	0.206	0.377	0.366	0.000	0.479	3.745

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	36	24	25	62	0	35	25
N.S.	1	1.00	1.44	0.96	1.00	2.48	0.00	1.40	1.00
time (sec)	N/A	0.054	0.075	0.260	0.341	0.354	0.000	0.482	3.634

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	107	68	83	142	0	118	149
N.S.	1	1.00	1.78	1.13	1.38	2.37	0.00	1.97	2.48
time (sec)	N/A	0.047	0.036	0.301	0.370	0.361	0.000	0.495	3.716

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	46	50	122	0	62	49
N.S.	1	1.00	1.26	0.81	0.88	2.14	0.00	1.09	0.86
time (sec)	N/A	0.062	0.275	0.260	0.339	0.342	0.000	0.548	3.672

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	151	90	123	213	0	177	211
N.S.	1	1.00	1.54	0.92	1.26	2.17	0.00	1.81	2.15
time (sec)	N/A	0.065	0.046	0.306	0.378	0.361	0.000	0.537	3.846

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	104	66	72	174	0	84	70
N.S.	1	1.00	1.20	0.76	0.83	2.00	0.00	0.97	0.80
time (sec)	N/A	0.075	0.594	0.262	0.342	0.341	0.000	0.500	3.930

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	240	140	128	137	0	5784	127
N.S.	1	1.00	2.12	1.24	1.13	1.21	0.00	51.19	1.12
time (sec)	N/A	0.127	3.771	0.149	0.546	0.375	0.000	2.749	3.820

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	152	113	104	126	0	34422	174
N.S.	1	1.00	1.25	0.93	0.85	1.03	0.00	282.15	1.43
time (sec)	N/A	0.092	1.011	0.162	0.413	0.346	0.000	15.300	6.706

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	162	109	82	101	0	1061	75
N.S.	1	1.00	2.13	1.43	1.08	1.33	0.00	13.96	0.99
time (sec)	N/A	0.076	2.672	0.159	0.576	0.340	0.000	0.717	3.688

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	83	67	90	0	2837	93
N.S.	1	1.00	1.63	1.22	0.99	1.32	0.00	41.72	1.37
time (sec)	N/A	0.054	0.499	0.160	0.357	0.345	0.000	1.096	4.042

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	97	56	60	102	0	74	125
N.S.	1	1.00	2.26	1.30	1.40	2.37	0.00	1.72	2.91
time (sec)	N/A	0.038	0.266	0.234	0.334	0.349	0.000	0.570	3.721

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	91	38	39	96	0	51	44
N.S.	1	1.00	2.17	0.90	0.93	2.29	0.00	1.21	1.05
time (sec)	N/A	0.032	0.628	0.227	0.332	0.342	0.000	0.573	3.641

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	250	101	122	230	0	172	292
N.S.	1	1.00	2.63	1.06	1.28	2.42	0.00	1.81	3.07
time (sec)	N/A	0.079	1.987	0.238	0.331	0.408	0.000	0.589	3.845

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	80	69	174	0	91	72
N.S.	1	1.00	1.61	1.01	0.87	2.20	0.00	1.15	0.91
time (sec)	N/A	0.048	1.513	0.209	0.350	0.356	0.000	0.589	3.787

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	994	145	187	333	0	269	378
N.S.	1	1.00	6.02	0.88	1.13	2.02	0.00	1.63	2.29
time (sec)	N/A	0.111	6.215	0.275	0.362	0.395	0.000	0.630	3.909

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	119	104	240	0	131	107
N.S.	1	1.00	0.93	0.98	0.85	1.97	0.00	1.07	0.88
time (sec)	N/A	0.071	1.629	0.241	0.416	0.346	0.000	0.596	4.060

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	771	184	173	188	0	72200	291
N.S.	1	1.00	3.76	0.90	0.84	0.92	0.00	352.20	1.42
time (sec)	N/A	0.135	6.317	0.205	0.352	0.347	0.000	168.918	6.436

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	203	163	113	149	0	2590	151
N.S.	1	1.00	1.97	1.58	1.10	1.45	0.00	25.15	1.47
time (sec)	N/A	0.100	4.665	0.171	0.673	0.354	0.000	1.324	3.695

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	637	144	128	144	0	14636	193
N.S.	1	1.00	4.79	1.08	0.96	1.08	0.00	110.05	1.45
time (sec)	N/A	0.082	6.168	0.207	0.428	0.347	0.000	9.236	5.799

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	241	107	111	148	0	144	278
N.S.	1	1.00	2.80	1.24	1.29	1.72	0.00	1.67	3.23
time (sec)	N/A	0.065	2.370	0.258	0.372	0.406	0.000	0.787	4.209

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	126	55	56	127	0	70	62
N.S.	1	1.00	1.97	0.86	0.88	1.98	0.00	1.09	0.97
time (sec)	N/A	0.037	1.054	0.233	0.431	0.342	0.000	0.774	3.662

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	897	140	171	299	0	304	581
N.S.	1	1.00	6.36	0.99	1.21	2.12	0.00	2.16	4.12
time (sec)	N/A	0.101	6.194	0.271	0.633	0.403	0.000	0.808	3.966

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	212	106	98	237	0	133	103
N.S.	1	1.00	1.88	0.94	0.87	2.10	0.00	1.18	0.91
time (sec)	N/A	0.063	2.304	0.260	0.355	0.352	0.000	0.809	3.719

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	198	250	427	0	373	698
N.S.	1	1.00	5.37	0.86	1.09	1.86	0.00	1.63	3.05
time (sec)	N/A	0.150	6.204	0.296	0.350	0.439	0.000	0.841	4.038

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	515	165	142	343	0	189	146
N.S.	1	1.00	3.08	0.99	0.85	2.05	0.00	1.13	0.87
time (sec)	N/A	0.094	1.899	0.267	0.458	0.371	0.000	0.855	3.877

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	267	218	224	0	0	319
N.S.	1	1.00	3.70	0.97	0.79	0.81	0.00	0.00	1.16
time (sec)	N/A	0.178	6.318	0.229	0.322	0.348	0.000	0.000	7.213

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	263	250	154	186	0	3931	161
N.S.	1	1.00	1.89	1.80	1.11	1.34	0.00	28.28	1.16
time (sec)	N/A	0.119	6.291	0.196	0.530	0.356	0.000	2.105	3.794

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	383	217	166	176	0	22530	268
N.S.	1	1.00	2.13	1.21	0.92	0.98	0.00	125.17	1.49
time (sec)	N/A	0.117	5.131	0.222	0.316	0.397	0.000	8.199	7.285

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	352	170	139	175	0	193	496
N.S.	1	1.00	2.98	1.44	1.18	1.48	0.00	1.64	4.20
time (sec)	N/A	0.084	4.971	0.283	0.477	0.387	0.000	0.990	4.144

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	162	79	72	159	0	86	81
N.S.	1	1.00	1.95	0.95	0.87	1.92	0.00	1.04	0.98
time (sec)	N/A	0.047	1.265	0.221	0.307	0.344	0.000	0.989	3.663

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	157	188	346	0	300	670
N.S.	1	1.00	7.01	0.98	1.17	2.15	0.00	1.86	4.16
time (sec)	N/A	0.120	6.218	0.303	0.309	0.416	0.000	1.023	4.069

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	188	133	120	267	0	161	132
N.S.	1	1.00	1.37	0.97	0.88	1.95	0.00	1.18	0.96
time (sec)	N/A	0.084	3.779	0.279	0.306	0.375	0.000	1.038	3.761

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	241	304	547	0	479	857
N.S.	1	1.00	5.44	0.88	1.11	2.00	0.00	1.75	3.13
time (sec)	N/A	0.190	6.304	0.329	0.337	0.473	0.000	1.054	4.171

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	233	218	171	386	0	235	181
N.S.	1	1.00	1.20	1.12	0.88	1.99	0.00	1.21	0.93
time (sec)	N/A	0.119	4.330	0.293	0.334	0.367	0.000	1.073	3.827

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	660	327	387	697	0	647	990
N.S.	1	1.00	1.64	0.81	0.96	1.73	0.00	1.61	2.46
time (sec)	N/A	0.240	6.280	0.433	0.341	0.491	0.000	1.217	4.035

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	289	358	658	370	0	464	683
N.S.	1	1.00	1.05	1.31	2.40	1.35	0.00	1.69	2.49
time (sec)	N/A	0.275	3.370	0.399	0.570	0.372	0.000	0.498	6.830

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	249	208	280	216	0	334	313
N.S.	1	1.00	1.58	1.32	1.77	1.37	0.00	2.11	1.98
time (sec)	N/A	0.230	2.959	0.299	0.560	0.370	0.000	0.454	4.269

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	202	364	261	0	241	324
N.S.	1	1.00	0.83	1.20	2.17	1.55	0.00	1.43	1.93
time (sec)	N/A	0.168	1.585	0.317	0.566	0.350	0.000	0.468	6.497

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	170	122	144	122	0	184	147
N.S.	1	1.00	1.81	1.30	1.53	1.30	0.00	1.96	1.56
time (sec)	N/A	0.121	0.825	0.261	0.615	0.388	0.000	0.446	3.865

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	101	141	185	0	118	110
N.S.	1	1.00	0.88	1.12	1.57	2.06	0.00	1.31	1.22
time (sec)	N/A	0.072	0.380	0.274	0.629	0.395	0.000	0.478	3.913

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	63	107	183	0	94	174
N.S.	1	1.00	1.14	0.95	1.62	2.77	0.00	1.42	2.64
time (sec)	N/A	0.093	0.129	0.312	0.637	0.408	0.000	0.478	4.092

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	48	47	95	0	60	39
N.S.	1	1.00	0.94	0.96	0.94	1.90	0.00	1.20	0.78
time (sec)	N/A	0.040	0.147	0.250	0.340	0.361	0.000	0.428	3.737

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	179	140	215	270	0	209	764
N.S.	1	1.00	1.47	1.15	1.76	2.21	0.00	1.71	6.26
time (sec)	N/A	0.213	0.861	0.375	0.559	0.391	0.000	0.473	4.448

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	96	97	208	0	144	102
N.S.	1	1.00	0.88	0.89	0.90	1.93	0.00	1.33	0.94
time (sec)	N/A	0.073	0.528	0.277	0.327	0.413	0.000	0.463	3.843

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	165	168	385	0	251	167
N.S.	1	1.00	0.89	0.98	0.99	2.28	0.00	1.49	0.99
time (sec)	N/A	0.110	2.197	0.299	0.305	0.379	0.000	0.462	4.329

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	603	383	799	619	0	735	757
N.S.	1	1.00	2.03	1.29	2.69	2.08	0.00	2.47	2.55
time (sec)	N/A	0.796	6.578	0.487	0.561	0.423	0.000	0.542	5.543

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	373	269	507	444	0	513	481
N.S.	1	1.00	1.72	1.24	2.34	2.05	0.00	2.36	2.22
time (sec)	N/A	0.450	4.186	0.382	0.555	0.404	0.000	0.554	4.820

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	246	171	293	292	0	263	255
N.S.	1	1.00	1.66	1.16	1.98	1.97	0.00	1.78	1.72
time (sec)	N/A	0.218	3.523	0.406	0.525	0.423	0.000	0.530	4.056

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	109	67	74	293	0	74	79
N.S.	1	1.00	1.51	0.93	1.03	4.07	0.00	1.03	1.10
time (sec)	N/A	0.047	0.427	0.315	0.323	0.378	0.000	0.516	3.842

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	244	127	144	442	0	203	150
N.S.	1	1.00	1.74	0.91	1.03	3.16	0.00	1.45	1.07
time (sec)	N/A	0.090	2.830	0.371	0.308	0.376	0.000	0.507	3.915

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	589	205	225	787	0	332	237
N.S.	1	1.00	2.69	0.94	1.03	3.59	0.00	1.52	1.08
time (sec)	N/A	0.153	6.264	0.370	0.305	0.388	0.000	0.583	5.015

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	683	466	1088	932	0	923	1068
N.S.	1	1.00	1.79	1.22	2.85	2.44	0.00	2.42	2.80
time (sec)	N/A	1.284	6.714	0.565	0.600	0.465	0.000	0.686	5.842

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	501	331	744	705	0	588	717
N.S.	1	1.00	1.76	1.16	2.61	2.47	0.00	2.06	2.52
time (sec)	N/A	0.748	6.513	0.530	0.552	0.407	0.000	0.671	5.335

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	316	236	463	526	0	482	433
N.S.	1	1.00	1.53	1.15	2.25	2.55	0.00	2.34	2.10
time (sec)	N/A	0.325	4.223	0.462	0.540	0.385	0.000	0.676	4.623

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	241	85	108	565	0	113	99
N.S.	1	1.00	2.54	0.89	1.14	5.95	0.00	1.19	1.04
time (sec)	N/A	0.054	2.864	0.385	0.332	0.444	0.000	0.599	3.869

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	456	158	192	811	0	237	200
N.S.	1	1.00	2.56	0.89	1.08	4.56	0.00	1.33	1.12
time (sec)	N/A	0.110	3.927	0.410	0.321	0.404	0.000	0.604	4.367

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	494	246	281	1018	0	382	297
N.S.	1	1.00	1.86	0.93	1.06	3.84	0.00	1.44	1.12
time (sec)	N/A	0.174	4.988	0.447	0.349	0.414	0.000	0.628	5.321

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	564	425	997	1053	0	902	962
N.S.	1	1.00	1.54	1.16	2.72	2.88	0.00	2.46	2.63
time (sec)	N/A	1.261	5.998	0.718	0.609	0.478	0.000	0.802	5.662

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	395	288	662	802	0	642	597
N.S.	1	1.00	1.50	1.09	2.51	3.04	0.00	2.43	2.26
time (sec)	N/A	0.473	3.958	0.610	0.562	0.459	0.000	0.788	5.088

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	259	103	140	874	0	129	131
N.S.	1	1.00	2.23	0.89	1.21	7.53	0.00	1.11	1.13
time (sec)	N/A	0.061	2.337	0.415	0.315	0.410	0.000	0.662	3.976

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	528	184	228	1235	0	222	232
N.S.	1	1.00	2.58	0.90	1.11	6.02	0.00	1.08	1.13
time (sec)	N/A	0.128	2.208	0.458	0.319	0.428	0.000	0.683	5.058

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	673	280	325	1536	0	428	337
N.S.	1	1.00	2.24	0.93	1.08	5.12	0.00	1.43	1.12
time (sec)	N/A	0.206	1.825	0.496	0.359	0.479	0.000	0.733	5.647

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	50	55	0	44	38
N.S.	1	1.00	1.58	1.00	1.92	2.12	0.00	1.69	1.46
time (sec)	N/A	0.054	0.049	0.133	0.528	0.370	0.000	0.441	3.881

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	2.824	0.408	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	1.339	0.345	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.296	0.268	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.471	15.045	0.552	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	1.822	3.557	0.313	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	910	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	6.634	1.386	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	270	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	1.251	1.145	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.937	0.283	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	78	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.287	0.286	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	1.431	3.181	0.877	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.637	2.424	0.013	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.348	1.696	0.289	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	1.228	14.835	0.300	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [22] had the largest ratio of [21]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	13	0.308
2	A	9	7	1.00	13	0.538
3	A	5	4	1.00	13	0.308
4	A	8	6	1.00	11	0.546
5	A	8	7	1.00	11	0.636
6	A	3	2	1.00	13	0.154
7	A	8	7	1.00	13	0.538
8	A	4	3	1.00	13	0.231
9	A	9	8	1.00	13	0.615
10	A	4	3	1.00	13	0.231
11	A	8	5	1.00	19	0.263
12	A	6	4	1.00	19	0.210
13	A	8	5	1.00	19	0.263
14	A	5	4	1.00	19	0.210
15	A	6	5	1.00	17	0.294
16	A	4	2	1.00	17	0.118
17	A	3	1	1.00	19	0.053
18	A	7	6	1.00	19	0.316
19	A	3	1	1.00	19	0.053
20	A	9	6	1.00	19	0.316
21	A	3	1	1.00	19	0.053
22	A	8	6	1.00	21	0.286
23	A	11	7	1.00	21	0.333
24	A	6	6	1.00	21	0.286
25	A	9	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	4	1.00	19	0.210
27	A	3	2	1.00	21	0.095
28	A	10	7	1.00	21	0.333
29	A	3	2	1.00	21	0.095
30	A	13	9	1.00	21	0.429
31	A	3	2	1.00	21	0.095
32	A	16	8	1.00	21	0.381
33	A	7	6	1.00	21	0.286
34	A	13	8	1.00	19	0.421
35	A	8	5	1.00	19	0.263
36	A	3	2	1.00	21	0.095
37	A	12	7	1.00	21	0.333
38	A	3	2	1.00	21	0.095
39	A	17	9	1.00	21	0.429
40	A	3	2	1.00	21	0.095
41	A	19	8	1.00	21	0.381
42	A	7	6	1.00	21	0.286
43	A	16	9	1.00	19	0.474
44	A	10	5	1.00	19	0.263
45	A	3	2	1.00	21	0.095
46	A	14	9	1.00	21	0.429
47	A	3	2	1.00	21	0.095
48	A	21	9	1.00	21	0.429
49	A	3	2	1.00	21	0.095
50	A	25	9	1.00	21	0.429
51	A	13	9	1.00	21	0.429
52	A	8	6	1.00	21	0.286
53	A	10	9	1.00	21	0.429
54	A	7	6	1.00	21	0.286
55	A	6	6	1.00	19	0.316
56	A	6	5	1.00	19	0.263
57	A	3	2	1.00	21	0.095
58	A	15	11	1.00	21	0.524
59	A	3	2	1.00	21	0.095
60	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	9	6	1.00	21	0.286
62	A	8	6	1.00	21	0.286
63	A	7	6	1.00	21	0.286
64	A	3	2	1.00	21	0.095
65	A	3	2	1.00	21	0.095
66	A	3	2	1.00	21	0.095
67	A	9	6	1.00	21	0.286
68	A	8	6	1.00	21	0.286
69	A	7	6	1.00	21	0.286
70	A	3	2	1.00	21	0.095
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	8	6	1.00	21	0.286
74	A	7	6	1.00	21	0.286
75	A	3	2	1.00	21	0.095
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	6	5	1.00	9	0.556
79	A	8	5	1.00	21	0.238
80	A	6	5	1.00	21	0.238
81	A	5	4	1.00	19	0.210
82	A	14	6	1.00	21	0.286
83	A	0	0	0.00	0	0.000
84	A	7	4	1.00	21	0.190
85	A	6	4	1.00	21	0.190
86	A	2	2	1.00	21	0.095
87	A	4	4	1.00	21	0.190
88	A	0	0	0.00	0	0.000
89	A	0	0	0.00	0	0.000
90	A	0	0	0.00	0	0.000
91	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{ix}{16}-\frac{1}{32(i-\tan(x))^2}-\frac{i}{8(i-\tan(x))}+\frac{i}{24(i+\tan(x))^3}-\frac{5}{32(i+\tan(x))^2}-\frac{3i}{16(i+\tan(x))}$$

```
[Out] -1/16*I*x-1/32/(I-tan(x))^2-1/8*I/(I-tan(x))+1/24*I/(I+tan(x))^3-5/32/(I+ta
n(x))^2-3/16*I/(I+tan(x))
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3597, 862, 90, 209}

$$-\frac{ix}{16}-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (-1/16*I)*x - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I/24)/(I + Tan[
x])^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \text{Subst}\left(\int \frac{x^4}{(i+x)\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^4}{(-i+x)^3(i+x)^4}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} - \frac{i}{8(i+x)^4} + \frac{5}{16(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{16(1+$$
$$= -\frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x$$
$$= -\frac{ix}{16} - \frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{}{16(i+$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.86

$$\frac{\sec(x)(-56i\cos(x) - 9i\cos(3x) + i\cos(5x) + 24\text{ArcTan}(\tan(x))(\cos(x) - i\sin(x)) - 32\sin(x) - 27\sin(3x) + 5\sin(5x))}{384(i+\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(I + Tan[x]),x]
[Out] (Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]]*(
Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x]))/(384*(I + Tan[x
]))
```

Maple [A]
time = 0.11, size = 66, normalized size = 0.85

method	result	size
risch	$-\frac{ix}{16} - \frac{e^{6ix}}{192} + \frac{\cos(4x)}{32} + \frac{i\sin(4x)}{64} - \frac{5\cos(2x)}{64} + \frac{i\sin(2x)}{64}$	39
default	$\frac{i}{24(i+\tan(x))^3} - \frac{3i}{16(i+\tan(x))} - \frac{5}{32(i+\tan(x))^2} + \frac{\ln(i+\tan(x))}{32} + \frac{i}{8\tan(x)-8i} - \frac{1}{32(\tan(x)-i)^2} - \frac{\ln(\tan(x)-i)}{32}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*I/(I+tan(x))^3-3/16*I/(I+tan(x))-5/32/(I+tan(x))^2+1/32*ln(I+tan(x))+1
/8*I/(tan(x)-I)-1/32/(tan(x)-I)^2-1/32*ln(tan(x)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.32, size = 39, normalized size = 0.50

$$\frac{1}{384}\left(-24i\,xe^{(4i\,x)}-2\,e^{(10i\,x)}+9\,e^{(8i\,x)}-12\,e^{(6i\,x)}-18\,e^{(2i\,x)}+3\right)e^{(-4i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*e
^(2*I*x) + 3)*e^(-4*I*x)
```

Sympy [A]
time = 0.08, size = 51, normalized size = 0.65

$$-\frac{ix}{16}-\frac{e^{6ix}}{192}+\frac{3e^{4ix}}{128}-\frac{e^{2ix}}{32}-\frac{3e^{-2ix}}{64}+\frac{e^{-4ix}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(I+tan(x)),x)
```

```
[Out] -I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x)
/64 + exp(-4*I*x)/128
```

Giac [A]
time = 0.44, size = 53, normalized size = 0.68

$$-\frac{3i\,\tan\left(x\right)^{4}+21\,\tan\left(x\right)^{3}+13i\,\tan\left(x\right)^{2}+11\,\tan\left(x\right)+8i}{48\left(\tan\left(x\right)+i\right)^{3}\left(\tan\left(x\right)-i\right)^{2}}+\frac{1}{32}\,\log\left(\tan\left(x\right)+i\right)-\frac{1}{32}\,\log\left(\tan\left(x\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan(
x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)
```

Mupad [B]
time = 3.71, size = 49, normalized size = 0.63

$$-\frac{x\,1\mathrm{i}}{16}+\frac{\frac{\tan(x)^4\,1\mathrm{i}}{16}+\frac{7\tan(x)^3}{16}+\frac{\tan(x)^2\,13\mathrm{i}}{48}+\frac{11\tan(x)}{48}+\frac{1}{6}\mathrm{i}}{\left(\tan\left(x\right)+1\mathrm{i}\right)^3\,\left(1+\tan\left(x\right)\,1\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(tan(x) + 1i),x)
```

```
[Out] ((11*tan(x))/48 + (tan(x)^2*13i)/48 + (7*tan(x)^3)/16 + (tan(x)^4*1i)/16 +
1i/6)/((tan(x) + 1i)^3*(tan(x)*1i + 1)^2) - (x*1i)/16
```

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=29

$$\frac{1}{3} i \cos^3(x) - \frac{1}{5} i \cos^5(x) + \frac{\sin^5(x)}{5}$$

```
[Out] 1/3*I*cos(x)^3-1/5*I*cos(x)^5+1/5*sin(x)^5
```

Rubi [A]
time = 0.10, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3599, 3187, 3186, 2645, 14, 2644, 30}

$$\frac{\sin^5(x)}{5} - \frac{1}{5} i \cos^5(x) + \frac{1}{3} i \cos^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin^3(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cos(x)(\cos(x)+i\sin(x))\sin^3(x)\,dx\right) \\
&= -\left(i\int \left(\cos^2(x)\sin^3(x)+i\cos(x)\sin^4(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin^3(x)\,dx\right)+\int \cos(x)\sin^4(x)\,dx \\
&= i\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(x)\right)+\text{Subst}\left(\int x^4\,dx,x,\sin(x)\right) \\
&= \frac{\sin^5(x)}{5}+i\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(x)\right) \\
&= \frac{1}{3}i\cos^3(x)-\frac{1}{5}i\cos^5(x)+\frac{\sin^5(x)}{5}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.76

$$\frac{1}{8}i\cos(x)+\frac{1}{48}i\cos(3x)-\frac{1}{80}i\cos(5x)+\frac{\sin(x)}{8}-\frac{1}{16}\sin(3x)+\frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16 +
 Sin[5*x]/80
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(21) = 42$.
time = 0.10, size = 81, normalized size = 2.79

method	result	si
risch	$-\frac{ie^{5ix}}{80}+\frac{ie^{-ix}}{8}+\frac{i\cos(3x)}{48}-\frac{\sin(3x)}{16}$	3
default	$-\frac{i}{4(\tan(\frac{x}{2})-i)^2}+\frac{1}{6(\tan(\frac{x}{2})-i)^3}+\frac{1}{8\tan(\frac{x}{2})-8i}+\frac{i}{(\tan(\frac{x}{2})+i)^4}+\frac{2}{5(\tan(\frac{x}{2})+i)^5}-\frac{2}{3(\tan(\frac{x}{2})+i)^3}-\frac{1}{8(\tan(\frac{x}{2})+i)}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/(tan(1/2*x)-I)^2+1/6/(tan(1/2*x)-I)^3+1/8/(tan(1/2*x)-I)+I/(tan(1/2*
x)+I)^4+2/5/(tan(1/2*x)+I)^5-2/3/(tan(1/2*x)+I)^3-1/8/(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.32, size = 26, normalized size = 0.90

$$\frac{1}{240}\left(-3i\,e^{(8i\,x)}+10i\,e^{(6i\,x)}+30i\,e^{(2i\,x)}-5i\right)e^{(-3i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)
```

Sympy [A]
time = 0.10, size = 37, normalized size = 1.28

$$-\frac{ie^{5ix}}{80}+\frac{ie^{3ix}}{24}+\frac{ie^{-ix}}{8}-\frac{ie^{-3ix}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(I+tan(x)),x)
```

```
[Out] -I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.43, size = 71, normalized size = 2.45

$$-\frac{-3i\tan\left(\frac{1}{2}x\right)^2 - 12\tan\left(\frac{1}{2}x\right) + 5i}{24\left(-i\tan\left(\frac{1}{2}x\right) - 1\right)^3} - \frac{15\tan\left(\frac{1}{2}x\right)^4 + 60i\tan\left(\frac{1}{2}x\right)^3 - 10\tan\left(\frac{1}{2}x\right)^2 - 20i\tan\left(\frac{1}{2}x\right) + 7}{120\left(\tan\left(\frac{1}{2}x\right) + i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/1
20*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*x)
 + 7)/(tan(1/2*x) + I)^5
```

Mupad [B]
time = 3.87, size = 59, normalized size = 2.03

$$-\frac{4\left(-\tan\left(\frac{x}{2}\right)^4 15\mathrm{i} + 6\tan\left(\frac{x}{2}\right)^3 + \tan\left(\frac{x}{2}\right)^2 2\mathrm{i} + 2\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)}{15\left(-1 + \tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^5\left(1 + \tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(tan(x) + 1i),x)
```

```
[Out] -(4*(2*tan(x/2) + tan(x/2)^2*2i + 6*tan(x/2)^3 - tan(x/2)^4*15i + 1i))/(15*
(tan(x/2)*1i - 1)^5*(tan(x/2)*1i + 1)^3)
```

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}$$

```
[Out] -1/8*I*x-1/8*I/(I-tan(x))-1/8/(I+tan(x))^2-1/4*I/(I+tan(x))
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3597, 862, 90, 209}

$$-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(I + Tan[x]),x]
```

```
[Out] (-1/8*I)*x - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4)/(I + Tan[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(i+x)\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{(-i+x)^2(i+x)^3}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{8(-i+x)^2}+\frac{1}{4(i+x)^3}+\frac{i}{4(i+x)^2}-\frac{i}{8\left(1+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= -\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}-\frac{1}{8}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 39, normalized size = 0.78

$$-\frac{i(3+\cos(2x)-3i\sin(2x)+2\text{ArcTan}(\tan(x))(i+\tan(x)))}{16(i+\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(I + Tan[x]),x]
[Out] ((-1/16*I)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x])))
/(I + Tan[x])
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.94

method	result
risch	$-\frac{ix}{8}+\frac{e^{4ix}}{32}-\frac{\cos(2x)}{8}$
default	$-\frac{i}{4(i+\tan(x))}-\frac{1}{8(i+\tan(x))^2}+\frac{\ln(i+\tan(x))}{16}+\frac{i}{8\tan(x)-8i}-\frac{\ln(\tan(x)-i)}{16}$
norman	$\frac{-\frac{1}{4}+\frac{x\tan\left(\frac{x}{2}\right)}{2}+ix\tan(x)\tan\left(\frac{x}{2}\right)-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{i\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}-\frac{ix\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{x\tan(x)\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}-\frac{3i\tan(x)\left(\tan^4\left(\frac{x}{2}\right.\right.}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/(I+tan(x))-1/8/(I+tan(x))^2+1/16*ln(I+tan(x))+1/8*I/(tan(x)-I)-1/16*
ln(tan(x)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.32, size = 25, normalized size = 0.50

$$\frac{1}{32}\left(-4i\,xe^{(2i\,x)}+e^{(6i\,x)}-2\,e^{(4i\,x)}-2\right)e^{(-2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")
[Out] 1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.62

$$-\frac{ix}{8}+\frac{e^{4ix}}{32}-\frac{e^{2ix}}{16}-\frac{e^{-2ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(I+tan(x)),x)
[Out] -I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16
```

Giac [A]
time = 0.42, size = 41, normalized size = 0.82

$$-\frac{i\,\tan\left(x\right)^{2}+3\,\tan\left(x\right)+2i}{8\left(\tan\left(x\right)+i\right)^{2}\left(\tan\left(x\right)-i\right)}+\frac{1}{16}\,\log\left(\tan\left(x\right)+i\right)-\frac{1}{16}\,\log\left(\tan\left(x\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")
[Out] -1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*log
(tan(x) + I) - 1/16*log(tan(x) - I)
```

Mupad [B]
time = 3.75, size = 35, normalized size = 0.70

$$-\frac{x\,1\mathrm{i}}{8}+\frac{\frac{\tan(x)^2}{8}-\frac{\tan(x)\,3\mathrm{i}}{8}+\frac{1}{4}}{(\tan(x)+1\mathrm{i})^2\,(1+\tan(x)\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(tan(x) + 1i),x)
```

```
[Out] (tan(x)^2/8 - (tan(x)*3i)/8 + 1/4)/((tan(x) + 1i)^2*(tan(x)*1i + 1)) - (x*1
i)/8
```

3.4 $\int \frac{\sin(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=19

$$\frac{1}{3} i \cos^3(x) + \frac{\sin^3(x)}{3}$$

[Out] 1/3*I*cos(x)^3+1/3*sin(x)^3

Rubi [A]

time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3599, 3187, 3186, 2645, 30, 2644}

$$\frac{\sin^3(x)}{3} + \frac{1}{3} i \cos^3(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(I + Tan[x]),x]

[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -(i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)\,dx) \\
&= -\left(i\int \left(\cos^2(x)\sin(x)+i\cos(x)\sin^2(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin(x)\,dx\right)+\int \cos(x)\sin^2(x)\,dx \\
&= i\text{Subst}\left(\int x^2\,dx,x,\cos(x)\right)+\text{Subst}\left(\int x^2\,dx,x,\sin(x)\right) \\
&= \frac{1}{3}i\cos^3(x)+\frac{\sin^3(x)}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.74

$$\frac{1}{4}i\cos(x)+\frac{1}{12}i\cos(3x)+\frac{\sin(x)}{4}-\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(I + Tan[x]),x]
[Out] (I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.
time = 0.12, size = 47, normalized size = 2.47

method	result	size
risch	$\frac{i e^{3 i x}}{12}+\frac{i e^{-i x}}{4}$	18
default	$\frac{1}{2\tan\left(\frac{x}{2}\right)-2i}+\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2}{3\left(\tan\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{x}{2}\right)+i\right)}$	47
norman	$\frac{\frac{i\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{4i\left(\tan^{2}(x)\right)}{3}-\frac{\tan(x)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^{2}(x)\right)\tan\left(\frac{x}{2}\right)}{3}-\frac{4i\tan(x)\tan\left(\frac{x}{2}\right)}{3}+\frac{\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}+i}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan^{2}(x)+1\right)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(tan(1/2*x)-I)+I/(tan(1/2*x)+I)^2+2/3/(tan(1/2*x)+I)^3-1/2/(tan(1/2*x)+
I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.32, size = 14, normalized size = 0.74

$$\frac{1}{12}\left(i\,e^{(4i\,x)}+3i\right)e^{(-i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.89

$$\frac{ie^{3ix}}{12}+\frac{ie^{-ix}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x)
```

```
[Out] I*exp(3*I*x)/12 + I*exp(-I*x)/4
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.
time = 0.41, size = 33, normalized size = 1.74

$$-\frac{i}{2\left(-i\tan\left(\frac{1}{2}x\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}x\right)^{2}-1}{6\left(\tan\left(\frac{1}{2}x\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3
```

Mupad [B]
time = 3.83, size = 39, normalized size = 2.05

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)^{2}+\tan\left(\frac{x}{2}\right)\,2\mathrm{i}-1\right)}{3\,\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(tan(x) + 1i),x)
```

```
[Out] -(2*(tan(x/2)*2i + 3*tan(x/2)^2 - 1))/(3*(tan(x/2)*1i + 1)*(tan(x/2) + 1i)^
3)
```

3.5 $\int \frac{\csc(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=16

$$i\tanh^{-1}(\cos(x)) - i\cos(x) + \sin(x)$$

```
[Out] I*arctanh(cos(x))-I*cos(x)+sin(x)
```

Rubi [A]
time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3599, 3187, 3186, 2717, 2672, 327, 212}

$$\sin(x) - i\cos(x) + i\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(I + Tan[x]),x]
```

```
[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)}{i\cos(x)+\sin(x)}\,dx\\
&= -\left(i\int \cot(x)(\cos(x)+i\sin(x))\,dx\right)\\
&= -\left(i\int (i\cos(x)+\cos(x)\cot(x))\,dx\right)\\
&= -\left(i\int \cos(x)\cot(x)\,dx\right)+\int \cos(x)\,dx\\
&= \sin(x)+i\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(x)\right)\\
&= -i\cos(x)+\sin(x)+i\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(x)\right)\\
&= i\tanh^{-1}(\cos(x))-i\cos(x)+\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.94

$$-i\cos(x)+i\log\left(\cos\left(\frac{x}{2}\right)\right)-i\log\left(\sin\left(\frac{x}{2}\right)\right)+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(I + Tan[x]),x]
```

```
[Out] (-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

Maple [A]
time = 0.10, size = 21, normalized size = 1.31

method	result	size
default	$-i \ln \left(\tan \left(\frac{x}{2}\right)\right)+\frac{2}{\tan \left(\frac{x}{2}\right)+i}$	21
risch	$-i \mathrm{e}^{i x}-i \ln \left(\mathrm{e}^{i x}-1\right)+i \ln \left(\mathrm{e}^{i x}+1\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(tan(1/2*x))+2/(tan(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.47, size = 28, normalized size = 1.75

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+i}-i\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.
time = 0.33, size = 25, normalized size = 1.56

$$-i\,e^{(i\,x)}+i\,\log\left(e^{(i\,x)}+1\right)-i\,\log\left(e^{(i\,x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{\tan\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + I), x)
```

Giac [A]
time = 0.43, size = 22, normalized size = 1.38

$$-\frac{2i}{-i\tan\left(\frac{1}{2}x\right)+1}-i\log\left(-i\tan\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -2*I/(-I*tan(1/2*x) + 1) - I*log(-I*tan(1/2*x))
```

Mupad [B]
time = 3.80, size = 20, normalized size = 1.25

$$-\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i}+\frac{2}{\tan\left(\frac{x}{2}\right)+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1i)),x)
```

```
[Out] 2/(tan(x/2) + 1i) - log(tan(x/2))*1i
```

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=18

$$ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))$$

```
[Out] I*x+I*cot(x)+ln(cos(x))+ln(tan(x))
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3597, 46}

$$ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x^2(i+x)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-i-x} - \frac{i}{x^2} + \frac{1}{x}\right)\,dx, x, \tan(x)\right)\\
&= ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 0.83

$$ix + i\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Sin[x]]
```

Maple [A]
time = 0.13, size = 20, normalized size = 1.11

method	result	size
default	$-\ln\left(i+\tan\left(x\right)\right)+\ln\left(\tan\left(x\right)\right)+\frac{i}{\tan\left(x\right)}$	20
risch	$-\frac{2}{{\mathrm e}^{2 i x}-1}+\ln\left({\mathrm e}^{2 i x}-1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(I+tan(x))+ln(tan(x))+I/tan(x)
```

Maxima [A]
time = 0.34, size = 17, normalized size = 0.94

$$\frac{i}{\tan\left(x\right)} - \log\left(\tan\left(x\right) + i\right) + \log\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] I/tan(x) - log(tan(x) + I) + log(tan(x))
```

Fricas [A]
time = 0.32, size = 25, normalized size = 1.39

$$\frac{\left(e^{\left(2 i x\right)} - 1\right) \log\left(e^{\left(2 i x\right)} - 1\right) - 2}{e^{\left(2 i x\right)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(x\right)}{\tan\left(x\right) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**2/(tan(x) + I), x)
```

Giac [**A**]
time = 0.41, size = 18, normalized size = 1.00

$$\frac{i}{\tan(x)} - \log(\tan(x) + i) + \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="giac")
```

```
[Out] I/tan(x) - log(tan(x) + I) + log(abs(tan(x)))
```

Mupad [**B**]
time = 3.74, size = 19, normalized size = 1.06

$$\mathrm{atan}(2\tan(x) + 1\mathrm{i})\ 2\mathrm{i} + \frac{1\mathrm{i}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(tan(x) + 1i)),x)
```

```
[Out] atan(2*tan(x) + 1i)*2i + 1i/tan(x)
```

3.7 $\int \frac{\csc^3(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}i\tanh^{-1}(\cos(x))-\csc(x)+\frac{1}{2}i\cot(x)\csc(x)$$

```
[Out] -1/2*I*arctanh(cos(x))-csc(x)+1/2*I*cot(x)*csc(x)
```

Rubi [A]
time = 0.09, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.538$, Rules used = {3599, 3187, 3186, 2686, 8, 2691, 3855}

$$-\csc(x)-\frac{1}{2}i\tanh^{-1}(\cos(x))+\frac{1}{2}i\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] (-1/2*I)*ArcTanh[Cos[x]] - Csc[x] + (I/2)*Cot[x]*Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)\csc^2(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cot(x)\csc^2(x)(\cos(x)+i\sin(x))\,dx\right) \\
&= -\left(i\int \left(i\cot(x)\csc(x)+\cot^2(x)\csc(x)\right)\,dx\right) \\
&= -\left(i\int \cot^2(x)\csc(x)\,dx\right)+\int \cot(x)\csc(x)\,dx \\
&= \frac{1}{2}i\cot(x)\csc(x)+\frac{1}{2}i\int \csc(x)\,dx-\text{Subst}\left(\int 1\,dx,x,\csc(x)\right) \\
&= -\frac{1}{2}i\tanh^{-1}(\cos(x))-\csc(x)+\frac{1}{2}i\cot(x)\csc(x)
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 75 vs. $2(24) = 48$.
time = 0.03, size = 75, normalized size = 3.12

$$
-\frac{1}{2}\cot\left(\frac{x}{2}\right)+\frac{1}{8}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{2}i\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}i\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{8}i\sec^2\left(\frac{x}{2}\right)-\frac{1}{2}\tan\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] -1/2*Cot[x/2] + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2]
] - (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.
time = 0.16, size = 42, normalized size = 1.75

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{i\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{8\tan\left(\frac{x}{2}\right)^{2}}-\frac{1}{2\tan\left(\frac{x}{2}\right)}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$	42
risch	$-\frac{i\left(3\,\mathrm{e}^{3ix}-\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{2ix}-1\right)^{2}}+\frac{i\ln\left(\mathrm{e}^{ix}-1\right)}{2}-\frac{i\ln\left(\mathrm{e}^{ix}+1\right)}{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2-1/2/tan(1/2*x)+1/2*I*
ln(tan(1/2*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(16) = 32$.
time = 0.40, size = 59, normalized size = 2.46

$$-\frac{\left(\frac{4\sin\left(x\right)}{\cos\left(x\right)+1}-i\right)\left(\cos\left(x\right)+1\right)^{2}}{8\sin\left(x\right)^{2}}-\frac{\sin\left(x\right)}{2\left(\cos\left(x\right)+1\right)}-\frac{i\sin\left(x\right)^{2}}{8\left(\cos\left(x\right)+1\right)^{2}}+\frac{1}{2}i\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - I)*(cos(x) + 1)^2/sin(x)^2 - 1/2*sin(x)/(cos(
x) + 1) - 1/8*I*sin(x)^2/(cos(x) + 1)^2 + 1/2*I*log(sin(x)/(cos(x) + 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(16) = 32$.
time = 0.33, size = 73, normalized size = 3.04

$$\frac{\left(-i\,e^{(4\,i\,x)}+2i\,e^{(2\,i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4\,i\,x)}-2i\,e^{(2\,i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)-6i\,e^{(3\,i\,x)}+2i\,e^{(i\,x)}}{2\left(e^{(4\,i\,x)}-2\,e^{(2\,i\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) - 6*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{\tan(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**3/(tan(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.
time = 0.43, size = 46, normalized size = 1.92

$$-\frac{1}{8}i\,\tan\left(\frac{1}{2}\,x\right)^2 - \frac{6i\,\tan\left(\frac{1}{2}\,x\right)^2 + 4\,\tan\left(\frac{1}{2}\,x\right) - i}{8\,\tan\left(\frac{1}{2}\,x\right)^2} + \frac{1}{2}i\,\log\left(\tan\left(\frac{1}{2}\,x\right)\right) - \frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2 - 1/8*(6*I*tan(1/2*x)^2 + 4*tan(1/2*x) - I)/tan(1/2*x)^
2 + 1/2*I*log(tan(1/2*x)) - 1/2*tan(1/2*x)
```

Mupad [B]
time = 3.73, size = 41, normalized size = 1.71

$$-\frac{\tan\left(\frac{x}{2}\right)}{2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i}}{2} - \frac{2\tan\left(\frac{x}{2}\right) - \frac{1}{2}\mathrm{i}}{4\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/2 - tan(x/2)/2 - (2*tan(x/2) - 1i/2)/(4*tan(x/2)^2) - (t
an(x/2)^2*1i)/8
```

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3597, 862, 45}

$$-\frac{\cot^2(x)}{2}+\frac{1}{3}i\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] -1/2*Cot[x]^2 + (I/3)*Cot[x]^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^4(i+x)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{-i+x}{x^4}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{1}{x^3}\right)\,dx, x, \tan(x)\right)\\
&= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.53

$$
-\frac{1}{3}i\cot(x)-\frac{\csc^2(x)}{2}+\frac{1}{3}i\cot(x)\csc^2(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(I + Tan[x]),x]
[Out] (-1/3*I)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2
```

Maple [A]
time = 0.16, size = 15, normalized size = 0.79

method	result	size
default	$\frac{i}{3\tan(x)^3}-\frac{1}{2\tan(x)^2}$	15
risch	$\frac{4\,e^{4ix}-2\,e^{2ix}+\frac{2}{3}}{(e^{2ix}-1)^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*I/tan(x)^3-1/2/tan(x)^2
```

Maxima [A]
time = 0.41, size = 12, normalized size = 0.63

$$
-\frac{i\,(-3i\,\tan(x)-2)}{6\,\tan(x)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/6*I*(-3*I*tan(x) - 2)/tan(x)^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.
time = 0.31, size = 36, normalized size = 1.89

$$\frac{2\left(6\,e^{(4i\,x)}-3\,e^{(2i\,x)}+1\right)}{3\left(e^{(6i\,x)}-3\,e^{(4i\,x)}+3\,e^{(2i\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="fricas")
[Out] 2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x)
- 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^4(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(I+tan(x)),x)
[Out] Integral(csc(x)**4/(tan(x) + I), x)
```

Giac [A]
time = 0.44, size = 12, normalized size = 0.63

$$-\frac{3\,\tan(x)-2i}{6\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
[Out] -1/6*(3*tan(x) - 2*I)/tan(x)^3
```

Mupad [B]
time = 3.62, size = 13, normalized size = 0.68

$$\frac{\cot(x)^2\,\left(-3+\cot(x)\,2\mathrm{i}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(tan(x) + 1i)),x)
[Out] (cot(x)^2*(cot(x)*2i - 3))/6
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{8}i\tanh^{-1}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)$$

```
[Out] -1/8*I*arctanh(cos(x))-1/8*I*cot(x)*csc(x)-1/3*csc(x)^3+1/4*I*cot(x)*csc(x)
^3
```

Rubi [A]
time = 0.11, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3599, 3187, 3186, 2686, 30, 2691, 3853, 3855}

$$-\frac{\csc^3(x)}{3}-\frac{1}{8}i\tanh^{-1}(\cos(x))+\frac{1}{4}i\cot(x)\csc^3(x)-\frac{1}{8}i\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] (-1/8*I)*ArcTanh[Cos[x]] - (I/8)*Cot[x]*Csc[x] - Csc[x]^3/3 + (I/4)*Cot[x]*
Csc[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = \int \frac{\cot(x)\csc^4(x)}{i\cos(x)+\sin(x)}\,dx$$
$$= -\left(i\int \cot(x)\csc^4(x)(\cos(x)+i\sin(x))\,dx\right)$$
$$= -\left(i\int \left(i\cot(x)\csc^3(x)+\cot^2(x)\csc^3(x)\right)\,dx\right)$$
$$= -\left(i\int \cot^2(x)\csc^3(x)\,dx\right)+\int \cot(x)\csc^3(x)\,dx$$
$$= \frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{4}i\int \csc^3(x)\,dx - \text{Subst}\left(\int x^2\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{8}i\int \csc(x)\,dx$$
$$= -\frac{1}{8}i\tanh^{-1}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(40) = 80.
time = 0.03, size = 139, normalized size = 3.48

$$-\frac{1}{12}\cot\left(\frac{x}{2}\right)-\frac{1}{32}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{24}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}i\csc^4\left(\frac{x}{2}\right)-\frac{1}{8}i\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{8}i\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{32}i\sec^2\left(\frac{x}{2}\right)-\frac{1}{64}i\sec^4\left(\frac{x}{2}\right)-\frac{1}{12}\tan\left(\frac{x}{2}\right)-\frac{1}{24}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] -1/12*Cot[x/2] - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc[
x/2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 - (I
/64)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24
```

Maple [A]
time = 0.13, size = 58, normalized size = 1.45

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{i\left(\tan^4\left(\frac{x}{2}\right)\right)}{64}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{i}{64\tan\left(\frac{x}{2}\right)^4}-\frac{1}{24\tan\left(\frac{x}{2}\right)^3}-\frac{1}{8\tan\left(\frac{x}{2}\right)}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$	58
risch	$\frac{i\left(3\,e^{7ix}+53\,e^{5ix}-11\,e^{3ix}+3\,e^{ix}\right)}{12\left(e^{2ix}-1\right)^4}-\frac{i\ln\left(e^{ix}+1\right)}{8}+\frac{i\ln\left(e^{ix}-1\right)}{8}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tan(1/2*x)-1/64*I*tan(1/2*x)^4-1/24*tan(1/2*x)^3+1/64*I/tan(1/2*x)^4-1
/24/tan(1/2*x)^3-1/8/tan(1/2*x)+1/8*I*ln(tan(1/2*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.
time = 0.39, size = 83, normalized size = 2.08

$$-\frac{\left(\frac{8\sin(x)}{\cos(x)+1} + \frac{24\sin(x)^3}{(\cos(x)+1)^3} - 3i\right)(\cos(x)+1)^4}{192\sin(x)^4} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^3}{24(\cos(x)+1)^3} - \frac{i\sin(x)^4}{64(\cos(x)+1)^4} + \frac{1}{8}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*sin(x)/(cos(x) + 1) + 24*sin(x)^3/(cos(x) + 1)^3 - 3*I)*(cos(x) +
 1)^4/sin(x)^4 - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x) + 1)^3 - 1
/64*I*sin(x)^4/(cos(x) + 1)^4 + 1/8*I*log(sin(x)/(cos(x) + 1))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(26) = 52$.
time = 0.32, size = 123, normalized size = 3.08

$$-\frac{3\left(ie^{(8ix)} - 4ie^{(6ix)} + 6ie^{(4ix)} - 4ie^{(2ix)} + i\right)\log\left(e^{(ix)}+1\right) + 3\left(-ie^{(8ix)} + 4ie^{(6ix)} - 6ie^{(4ix)} + 4ie^{(2ix)} - i\right)\log\left(e^{(ix)}-1\right) - 6ie^{(7ix)} - 106ie^{(5ix)} + 22ie^{(3ix)} - 6ie^{(ix)}}{24\left(e^{(8ix)} - 4e^{(6ix)} + 6e^{(4ix)} - 4e^{(2ix)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(I*e^(8*I*x) - 4*I*e^(6*I*x) + 6*I*e^(4*I*x) - 4*I*e^(2*I*x) + I)*
log(e^(I*x) + 1) + 3*(-I*e^(8*I*x) + 4*I*e^(6*I*x) - 6*I*e^(4*I*x) + 4*I*e^
(2*I*x) - I)*log(e^(I*x) - 1) - 6*I*e^(7*I*x) - 106*I*e^(5*I*x) + 22*I*e^(3
*I*x) - 6*I*e^(I*x))/(e^(8*I*x) - 4*e^(6*I*x) + 6*e^(4*I*x) - 4*e^(2*I*x) +
 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**5/(tan(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.
time = 0.43, size = 62, normalized size = 1.55

$$-\frac{1}{64}i\tan\left(\frac{1}{2}x\right)^4 - \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 - \frac{50i\tan\left(\frac{1}{2}x\right)^4 + 24\tan\left(\frac{1}{2}x\right)^3 + 8\tan\left(\frac{1}{2}x\right) - 3i}{192\tan\left(\frac{1}{2}x\right)^4} + \frac{1}{8}i\log\left(\tan\left(\frac{1}{2}x\right)\right) - \frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*ta
n(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(tan(1/2*x)) - 1/8
*tan(1/2*x)
```

Mupad [B]
time = 3.70, size = 57, normalized size = 1.42

$$-\frac{\tan\left(\frac{x}{2}\right)}{8}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i}}{8}-\frac{2\tan\left(\frac{x}{2}\right)^3+\frac{2\tan\left(\frac{x}{2}\right)}{3}-\frac{1}{4}\mathrm{i}}{16\tan\left(\frac{x}{2}\right)^4}-\frac{\tan\left(\frac{x}{2}\right)^3}{24}-\frac{\tan\left(\frac{x}{2}\right)^4\,1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/8 - tan(x/2)/8 - ((2*tan(x/2))/3 + 2*tan(x/2)^3 - 1i/4)/
(16*tan(x/2)^4) - tan(x/2)^3/24 - (tan(x/2)^4*1i)/64
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3-1/4*cot(x)^4+1/5*I*cot(x)^5
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3597, 862, 76}

$$\frac{1}{5}i\cot^5(x)-\frac{\cot^4(x)}{4}+\frac{1}{3}i\cot^3(x)-\frac{\cot^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6/(I + Tan[x]),x]
```

```
[Out] -1/2*Cot[x]^2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(i+x)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{(-i+x)^2(i+x)}{x^6}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{i}{x^6}+\frac{1}{x^5}-\frac{i}{x^4}+\frac{1}{x^3}\right)dx, x, \tan(x)\right)$$
$$= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.11

$$-\frac{2}{15}i\cot(x)-\frac{1}{15}i\cot(x)\csc^2(x)-\frac{\csc^4(x)}{4}+\frac{1}{5}i\cot(x)\csc^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6/(I + Tan[x]),x]
[Out] ((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Csc
[x]^4
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.76

method	result	size
default	$-\frac{1}{4\tan(x)^4}-\frac{1}{2\tan(x)^2}+\frac{i}{3\tan(x)^3}+\frac{i}{5\tan(x)^5}$	28
risch	$-\frac{4(30\,e^{6ix}-10\,e^{4ix}+5\,e^{2ix}-1)}{15(e^{2ix}-1)^5}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^6/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] -1/4/tan(x)^4-1/2/tan(x)^2+1/3*I/tan(x)^3+1/5*I/tan(x)^5
```

Maxima [A]
time = 0.39, size = 24, normalized size = 0.65

$$\frac{i\left(30i\tan(x)^3+20\tan(x)^2+15i\tan(x)+12\right)}{60\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 0.31, size = 54, normalized size = 1.46

$$-\frac{4\left(30\,e^{(6i\,x)}-10\,e^{(4i\,x)}+5\,e^{(2i\,x)}-1\right)}{15\,(e^{(10i\,x)}-5\,e^{(8i\,x)}+10\,e^{(6i\,x)}-10\,e^{(4i\,x)}+5\,e^{(2i\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8*
I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{6}(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**6/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**6/(tan(x) + I), x)
```

Giac [A]
time = 0.44, size = 24, normalized size = 0.65

$$-\frac{30\,\tan(x)^{3}-20i\,\tan(x)^{2}+15\,\tan(x)-12i}{60\,\tan(x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5
```

Mupad [B]
time = 3.60, size = 27, normalized size = 0.73

$$\frac{\cot(x)^{5}\,1\mathrm{i}}{5}-\frac{\cot(x)^{4}}{4}+\frac{\cot(x)^{3}\,1\mathrm{i}}{3}-\frac{\cot(x)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6*(tan(x) + 1i)),x)
```

```
[Out] (cot(x)^3*1i)/3 - cot(x)^2/2 - cot(x)^4/4 + (cot(x)^5*1i)/5
```

3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin}{}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+2/3*a*cos(d*x+c)^3/d-1/5*a*cos(d*x+c
)^5/d-b*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3598, 2713, 2672, 308, 212}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*
d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*
d) - (b*Sin[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\sin^5(c+dx)+b\sin^5(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sin^5(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx\right.}{d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} + \frac{b\text{Subst}\left(\int \left(-\right.\right.}{}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\sin(c+dx)}{d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(}{5}$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 1.02

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a\cos(c+dx)}{8d} + \frac{5a\cos(3(c+dx))}{48d} - \frac{a\cos(5(c+dx))}{80d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*
x)])/(48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c +
 d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.19, size = 80, normalized size = 0.79

method	result

derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$\frac{11ie^{i(dx+c)}b}{16d}-\frac{5\,e^{i(dx+c)}a}{16d}-\frac{11ie^{-i(dx+c)}b}{16d}-\frac{5\,e^{-i(dx+c)}a}{16d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\cos(5dx}{80d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+b*(-1/5*sin(d*x+
c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.43, size = 91, normalized size = 0.90

$$-\frac{2\left(3\cos\left(dx+c\right)^{5}-10\cos\left(dx+c\right)^{3}+15\cos\left(dx+c\right)\right)a+\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right)^{3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)b}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(2*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a + (6*si
n(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*
x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Fricas [A]

time = 0.36, size = 97, normalized size = 0.96

$$-\frac{6\,a\cos\left(dx+c\right)^{5}-20\,a\cos\left(dx+c\right)^{3}+30\,a\cos\left(dx+c\right)-15\,b\log\left(\sin\left(dx+c\right)+1\right)+15\,b\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,b\cos\left(dx+c\right)^{4}-11\,b\cos\left(dx+c\right)^{2}+23\,b\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 30*a*cos(d*x + c) - 15*b*
log(sin(d*x + c) + 1) + 15*b*log(-sin(d*x + c) + 1) + 2*(3*b*cos(d*x + c)^4
 - 11*b*cos(d*x + c)^2 + 23*b)*sin(d*x + c))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 12356 vs. 2(93) = 186.
time = 1.66, size = 12356, normalized size = 122.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(15*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^10*tan(1/2*c)^10 - 15*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)
/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 16*a*tan(1/2*d*x)^10*t
an(1/2*c)^10 + 75*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 75*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 60*b*tan(1/2*d*x
)^10*tan(1/2*c)^9 + 75*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 75*b*log(2*(tan(1/2*d*x)^4*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 60*b*tan(1/
2*d*x)^9*tan(1/2*c)^10 + 80*a*tan(1/2*d*x)^10*tan(1/2*c)^8 + 80*a*tan(1/2*d
*x)^8*tan(1/2*c)^10 + 150*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
```

```
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 150*b*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 320*b*t
an(1/2*d*x)^10*tan(1/2*c)^7 + 375*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
+ 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 375*b*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 4
20*b*tan(1/2*d*x)^9*tan(1/2*c)^8 - 420*b*tan(1/2*d*x)^8*tan(1/2*c)^9 + 150*
b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^6*tan(1/2*c)^10 - 150*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 320*b*tan(1/2*d*x)^7*tan(1/2*c)^1
0 + 160*a*tan(1/2*d*x)^10*tan(1/2*c)^6 + 400*a*tan(1/2*d*x)^8*tan(1/2*c)^8
+ 160*a*tan(1/2*d*x)^6*tan(1/2*c)^10 + 150*b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 150*b*log(2*(
tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^
3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1
/2*c)^4 - 712*b*tan(1/2*d*x)^10*tan(1/2*c)^5 + ...
```

Mupad [B]
time = 3.80, size = 121, normalized size = 1.20

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\cos\left(c+d\,x\right)^5}{5\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{23\,b\,\sin\left(c+d\,x\right)}{15\,d}+\frac{11\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*cos(c + d*x)^3)
/(3*d) - (a*cos(c + d*x)^5)/(5*d) - (a*cos(c + d*x))/d - (23*b*sin(c + d*x)
)/(15*d) + (11*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^4*si
n(c + d*x))/(5*d)
```

3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}-\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d}$$

```
[Out] 3/8*a*x-b*ln(cos(d*x+c))/d-1/4*cos(d*x+c)*sin(d*x+c)^3*(a+b*tan(d*x+c))/d-1
/8*cos(d*x+c)*sin(d*x+c)*(3*a+4*b*tan(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {833, 649, 209, 266}

$$-\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*T
an[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/
(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx)}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4bx)}{(1+x^2)^2}\right.}{4\ldots}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+\ldots}{}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+\ldots}{}$$
$$= \frac{3ax}{8} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+\ldots}{4d}$$

Mathematica [A]

time = 0.09, size = 82, normalized size = 0.99

$$\frac{3a(c+dx)}{8d} - \frac{b\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] (3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c
+ d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]

time = 0.16, size = 73, normalized size = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$

risch	$ibx+\frac{3ax}{8}+\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16d}+\frac{i\mathrm{e}^{2i(dx+c)}a}{8d}+\frac{3\,\mathrm{e}^{-2i(dx+c)}b}{16d}-\frac{i\mathrm{e}^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}-\frac{b\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b*(-1/
4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.68, size = 87, normalized size = 1.05

$$\frac{3\,(dx+c)a+4\,b\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{5\,a\tan(dx+c)^3-8\,b\tan(dx+c)^2+3\,a\tan(dx+c)-6\,b}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8*
b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.89

$$-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-8\,b\cos\left(dx+c\right)^2+8\,b\log\left(-\cos\left(dx+c\right)\right)-\left(2\,a\cos\left(dx+c\right)^3-5\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - 8*b*cos(d*x + c)^2 + 8*b*log(-cos(d*x
+ c)) - (2*a*cos(d*x + c)^3 - 5*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. 2(77) = 154.
time = 0.59, size = 1066, normalized size = 12.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*
d*x*tan(d*x)^2*tan(c)^4 + 11*b*tan(d*x)^4*tan(c)^4 - 32*b*log(4*(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a*tan(d*x)^4*tan(
c)^3 - 32*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*ta
n(c)^4 + 12*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)
^2*tan(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 - 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x
*tan(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1))*tan(d*x)^4 + 20*a*tan(d*x)^4*tan(c) - 64*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 24*a*tan(d*x)^3*t
an(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(c)^4 + 20*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*x)^2 -
13*b*tan(d*x)^4 - 64*b*tan(d*x)^3*tan(c) + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x
)^2*tan(c)^2 - 64*b*tan(d*x)*tan(c)^3 - 13*b*tan(c)^4 - 32*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 20*a*tan(d*x)^3 - 24*a*tan
(d*x)^2*tan(c) - 32*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(c)^2 - 24*a*tan(d*x)*tan(c)^2 - 20*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2
- 32*b*tan(d*x)*tan(c) + 6*b*tan(c)^2 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1)) - 12*a*tan(d*x) - 12*a*tan(c) + 11*b)/(d*tan(d*x)^4*tan
(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 +
4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B]
time = 3.79, size = 155, normalized size = 1.87

$$\frac{3\,a\,x}{8}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}+\frac{3\,b}{4\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}-\frac{5\,a\tan(c+d\,x)^3}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}+\frac{b\tan(c+d\,x)^2}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}-\frac{3\,a\tan(c+d\,x)}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (b*log(tan(c + d*x)^2 + 1))/(2*d) + (3*b)/(4*d*(2*tan(c + d*x)^
2 + tan(c + d*x)^4 + 1)) - (5*a*tan(c + d*x)^3)/(8*d*(2*tan(c + d*x)^2 + ta
n(c + d*x)^4 + 1)) + (b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)
^4 + 1)) - (3*a*tan(c + d*x))/(8*d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-b*sin(d*x+c)/d-
1/3*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3598, 2713, 2672, 308, 212}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\sin^3(c+dx)+b\sin^3(c+dx)\tan(c+dx)\right)\,dx \\
&= a\int \sin^3(c+dx)\,dx+b\int \sin^3(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin\right.}{d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{b\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx\right.}{d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.03

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)
])/(12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.15, size = 60, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$

default	$\frac{-\frac{a\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^{3}(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$\frac{5i\mathrm{e}^{i(dx+c)}b}{8d}-\frac{3\,\mathrm{e}^{i(dx+c)}a}{8d}-\frac{5i\mathrm{e}^{-i(dx+c)}b}{8d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a}{8d}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}+\frac{a\cos(3dx+3}{12d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)+b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.36, size = 70, normalized size = 1.01

$$\frac{2\left(\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)a-\left(2\sin\left(dx+c\right)^{3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)+6\sin\left(dx+c\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d
```

Fricas [A]
time = 0.35, size = 74, normalized size = 1.07

$$\frac{2\,a\cos\left(dx+c\right)^{3}-6\,a\cos\left(dx+c\right)+3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(b\cos\left(dx+c\right)^{2}-4\,b\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3*
b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5350 vs. $2(65) = 130$.
time = 1.00, size = 5350, normalized size = 77.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
+ 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)
^6 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1/2*d*x)^6*tan(1/2*c)
^5 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^4*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 12*b*tan(1/2*d*x)^5*tan(1/2*c)
^6 + 12*a*tan(1/2*d*x)^6*tan(1/2*c)^4 + 12*a*tan(1/2*d*x)^4*tan(1/2*c)^6 +
9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^2 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c
```

```
)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 40*b*tan(1/2*d*x)^6*tan(1/2*c)^3 +
27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^4*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)^5*tan(1/2*c)^4
- 60*b*tan(1/2*d*x)^4*tan(1/2*c)^5 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6
- 40*b*tan(1/2*d*x)^3*tan(1/2*c)^6 - 12*a*tan(1/2*d*x)^6*tan(1/2*c)^2 - 96*
a*tan(1/2*d*x)^5*tan(1/2*c)^3 - 108*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - 96*a*ta
n(1/2*d*x)^3*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*b*log(2*(t
an(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 3*b*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6
- 12*b*tan(1/2*d*x)^6*tan(1/2*c) + 27*b*log(2*(...
```

Mupad [B]
time = 3.83, size = 87, normalized size = 1.26

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{4\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a*cos(c + d*x)^3)/(
3*d) - (a*cos(c + d*x))/d - (4*b*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=49

$$\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*a*x-b*ln(cos(d*x+c))/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {833, 649, 209, 266}

$$-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d}+\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c
 + d*x]))/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{a+2bx}{1+x^2}\,dx, x, \right.}{2d}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.}{2d}$$
$$= \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.14

$$\frac{a(c+dx)}{2d} - \frac{b\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*(-1/2*Cos[c + d*x]^2 + Log[Cos[c + d*x]]))/d - (a*
Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.14, size = 52, normalized size = 1.06

method	result	size
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
default	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
risch	$ibx+\frac{ax}{2}+\frac{e^{2i(dx+c)}b}{8d}+\frac{ie^{2i(dx+c)}a}{8d}+\frac{e^{-2i(dx+c)}b}{8d}-\frac{ie^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+b*(-1/2*sin(d*x+c)^2-ln(c
os(d*x+c))))

Maxima [A]
time = 0.55, size = 52, normalized size = 1.06

$$\frac{(dx+c)a+b\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d*
x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.96

$$\frac{adx+b\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b\log\left(-\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos(
d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(45) = 90.
time = 0.50, size = 413, normalized size = 8.43

$$\frac{2\,adx\tan\left(dx\right)^2\tan\left(c\right)^2-2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2+2\,adx\tan\left(dx\right)^2+2\,adx\tan\left(c\right)^2+b\tan\left(dx\right)^2\tan\left(c\right)^2-2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(dx\right)^2+2\,a\tan\left(dx\right)^2\tan\left(c\right)-2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan\left(c\right)^2+2\,a\tan\left(dx\right)\tan\left(c\right)^2+2\,adx-b\tan\left(dx\right)^2-4\,b\tan\left(dx\right)\tan\left(c\right)-b\tan\left(c\right)^2-2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)-2\,a\tan\left(dx\right)-2\,a\tan\left(c\right)+b}{4\left(d\tan\left(dx\right)^2\tan\left(c\right)^2+d\tan\left(dx\right)^2+d\tan\left(c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 +
 b*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(
c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c) - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b*tan(d*x)
^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2
+ d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B]
time = 3.82, size = 50, normalized size = 1.02

$$\frac{\frac{b\cos(c+d\,x)^2}{2}-\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{2}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+\frac{a\,d\,x}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] ((b*log(tan(c + d*x)^2 + 1))/2 + (b*cos(c + d*x)^2)/2 - (a*cos(c + d*x)*sin
(c + d*x))/2 + (a*d*x)/2)/d
```

3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=37

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d-b*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3598, 2718, 2672, 327, 212}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+b\tan(c+dx))\,dx &= \int (a\sin(c+dx)+b\sin(c+dx)\tan(c+dx))\,dx \\ &= a\int \sin(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx \\ &= -\frac{a\cos(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\ &= -\frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.30

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c)\cos(dx)}{d} + \frac{a\sin(c)\sin(dx)}{d} - \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

Maple [A]
time = 0.13, size = 40, normalized size = 1.08

method	result	size
derivativedivides	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
default	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
risch	$\frac{i e^{i(dx+c)} b}{2d} - \frac{e^{i(dx+c)} a}{2d} - \frac{i e^{-i(dx+c)} b}{2d} - \frac{e^{-i(dx+c)} a}{2d} - \frac{b\ln\left(e^{i(dx+c)}-i\right)}{d} + \frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*cos(d*x+c)+b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [**A**]
time = 0.33, size = 46, normalized size = 1.24

$$\frac{b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2\sin(dx+c)) - 2a\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 2
*a*cos(d*x + c))/d
```

Fricas [**A**]
time = 0.34, size = 49, normalized size = 1.32

$$-\frac{2a\cos(dx+c) - b\log(\sin(dx+c)+1) + b\log(-\sin(dx+c)+1) + 2b\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1)
 + 2*b*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(37) = 74$.
time = 0.53, size = 1236, normalized size = 33.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c
)^2 - 2*a*tan(1/2*d*x)^2 - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) - b*log(2*(
tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^
3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + 4*b*tan(1/2*d*x) +
4*b*tan(1/2*c) + 2*a)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d
*tan(1/2*c)^2 + d)
```

Mupad [B]
time = 3.82, size = 53, normalized size = 1.43

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(
c/2 + (d*x)/2)^2 + 1))
```

3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3598, 3855}

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\tan(c+dx))\,dx &= \int (a\csc(c+dx)+b\sec(c+dx))\,dx\\ &= a\int \csc(c+dx)\,dx+b\int \sec(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 2.00

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d
```

Maple [A]
time = 0.21, size = 40, normalized size = 1.54

method	result	size
derivativedivides	$\frac{a \ln(\csc(dx+c)-\cot(dx+c))+b \ln(\sec(dx+c)+\tan(dx+c))}{d}$	40
default	$\frac{a \ln(\csc(dx+c)-\cot(dx+c))+b \ln(\sec(dx+c)+\tan(dx+c))}{d}$	40
risch	$\frac{a \ln(e^{i(dx+c)}-1)}{d}-\frac{a \ln(e^{i(dx+c)}+1)}{d}-\frac{b \ln(e^{i(dx+c)}-i)}{d}+\frac{b \ln(e^{i(dx+c)}+i)}{d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*ln(csc(d*x+c)-cot(d*x+c))+b*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.38, size = 46, normalized size = 1.77

$$\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-2\,a\log(\cot(dx+c)+\csc(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x +
c) + csc(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.
time = 0.37, size = 58, normalized size = 2.23

$$-\frac{a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*lo
g(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx)) \csc(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x), x)
```

Giac [A]
time = 0.48, size = 49, normalized size = 1.88

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 3.75, size = 86, normalized size = 3.31

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b*atanh((b*cos(c/2 +
(d*x)/2) - a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/
2))))/d
```

3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$-\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {45}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 1.44

$$-\frac{a\cot(c+dx)}{d}-\frac{b(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [A]
time = 0.26, size = 24, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-\cot(dx+c)a+b\ln(\tan(dx+c))}{d}$	24
default	$\frac{-\cot(dx+c)a+b\ln(\tan(dx+c))}{d}$	24
risch	$-\frac{2ia}{d(\mathrm{e}^{2i(dx+c)}-1)}+\frac{b\ln(\mathrm{e}^{2i(dx+c)}-1)}{d}-\frac{b\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-cot(d*x+c)*a+b*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.34, size = 25, normalized size = 1.00

$$\frac{b\log\left(\tan\left(dx+c\right)\right)-\frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] (b*log(tan(d*x + c)) - a/tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.
time = 0.35, size = 62, normalized size = 2.48

$$-\frac{b\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)
*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.48, size = 35, normalized size = 1.40

$$\frac{b\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{b\tan(dx+c)+a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] (b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d
```

Mupad [B]
time = 3.63, size = 25, normalized size = 1.00

$$\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a\cot(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^2,x)
[Out] (b*log(tan(c + d*x)))/d - (a*cot(c + d*x))/d
```

3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-1/2*a*c
ot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3598, 3853, 3855, 2701, 327, 213}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Cos[c + d*x]])/d + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c +
 d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\csc^3(c+dx)+b\csc^2(c+dx)\sec(c+dx)\right)\,dx\\ &= a\int \csc^3(c+dx)\,dx+b\int \csc^2(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a\int \csc(c+dx)\,dx-\frac{b\text{Subst}\left(\int \frac{x}{-1-}\right.}{}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx}{2d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 107, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(c+dx)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c +
 d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.30, size = 68, normalized size = 1.13

method	result
derivativedivides	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$-\frac{i\left(ia\,e^{3i(dx+c)}+2b\,e^{3i(dx+c)}+ia\,e^{i(dx+c)}-2\,e^{i(dx+c)}b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+b*(-1/sin
(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.37, size = 83, normalized size = 1.38

$$\frac{a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,b\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d
*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(56) = 112$.
time = 0.36, size = 142, normalized size = 2.37

$$\frac{2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\sin\left(dx+c\right)+1\right)-2\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\sin\left(dx+c\right)+1\right)+4\,b\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)
+ (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^2
 - b)*log(sin(d*x + c) + 1) - 2*(b*cos(d*x + c)^2 - b)*log(-sin(d*x + c) +
1) + 4*b*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)\csc^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.
time = 0.50, size = 118, normalized size = 1.97

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*
b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 4*b*tan(1/2*d*x + 1/2*c) - (6*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x +
 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 3.72, size = 149, normalized size = 2.48

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{\frac{a}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{4\,b^{2}}{2\,a\,b-4\,b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,b-4\,b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2))/(4*d*tan(c/
2 + (d*x)/2)^2) - (b*tan(c/2 + (d*x)/2))/(2*d) - (2*b*atanh((4*b^2)/(2*a*b
- 4*b^2*tan(c/2 + (d*x)/2)) - (2*a*b*tan(c/2 + (d*x)/2))/(2*a*b - 4*b^2*tan
(c/2 + (d*x)/2))))/d + (a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-1/2*b*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d+b*ln(tan(d*x+c))/
d
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {780}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)
 + (b*Log[Tan[c + d*x]])/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)}{x^4}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 72, normalized size = 1.26

$$-\frac{2a\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b(\csc^2(c+dx)+2\log(\cos(c+dx))-2\log(\sin(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*(Csc
[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[Sin[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.26, size = 46, normalized size = 0.81

method	result	size
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot(dx+c)+b\left(-\frac{1}{2\sin(dx+c)^{2}}+\ln(\tan(dx+c))\right)}{d}$	46
default	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot(dx+c)+b\left(-\frac{1}{2\sin(dx+c)^{2}}+\ln(\tan(dx+c))\right)}{d}$	46
risch	$\frac{2b\,e^{4i(dx+c)}+4ia\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-\frac{4ia}{3}}{d\left(e^{2i(dx+c)}-1\right)^{3}}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c
))))
```

Maxima [A]
time = 0.34, size = 50, normalized size = 0.88

$$\frac{6\,b\log\left(\tan\left(dx+c\right)\right)-\frac{6\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/
tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(53) = 106.
time = 0.34, size = 122, normalized size = 2.14

$$-\frac{4\,a\cos\left(dx+c\right)^{3}+3\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-3\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^{2}+\frac{1}{4}\right)\sin\left(dx+c\right)-6\,a\cos\left(dx+c\right)-3\,b\sin\left(dx+c\right)}{6\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*sin
(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x
 + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))\csc^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)
```

Giac [A]
time = 0.55, size = 62, normalized size = 1.09

$$\frac{6\,b\log\left(\left|\tan\left(dx+c\right)\right|\right) - \frac{11\,b\tan\left(dx+c\right)^{3}+6\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2
 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d
```

Mupad [B]
time = 3.67, size = 49, normalized size = 0.86

$$\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{a\tan(c+d\,x)^2 + \frac{b\tan(c+d\,x)}{2} + \frac{a}{3}}{d\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/3 + (b*tan(c + d*x))/2 + a*tan(c + d*x)^2)/(d*
tan(c + d*x)^3)
```

3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx}{3d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3598, 3853, 3855, 2701, 308, 213}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
 + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\csc^5(c+dx)+b\csc^4(c+dx)\sec(c+dx)\right)\,dx \\
&= a\int \csc^5(c+dx)\,dx+b\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}(3a)\int \csc^3(c+dx)\,dx-\frac{b\text{Subst}\left(\int\right.}{} \\
&= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{8}(3a)\int \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\csc(c+dx)}{d}-\frac{3a\cot(c+dx)\csc(c+d}{8d} \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 151, normalized size = 1.54

$$
-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c
 + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*Lo
```

```
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.31, size = 90, normalized size = 0.92

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)-\right.}{d}$
default	$\frac{a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)-\right.}{d}$
risch	$-\frac{i\left(9ia\,e^{7i(dx+c)}+24b\,e^{7i(dx+c)}-33ia\,e^{5i(dx+c)}-104b\,e^{5i(dx+c)}-33ia\,e^{3i(dx+c)}+104b\,e^{3i(dx+c)}+9ia\,e^{i(dx+c)}-24\,e^{i(dx\right.}{12d\left(e^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot
(d*x+c)))+b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.38, size = 123, normalized size = 1.26

$$\frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-8\,b\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*(2
*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(si
n(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(90) = 180$.
time = 0.36, size = 213, normalized size = 2.17

$$\frac{18\,a\cos\left(dx+c\right)^3-30\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+24\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\sin\left(dx+c\right)+1\right)-24\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(-\sin\left(dx+c\right)+1\right)+16\left(3\,b\cos\left(dx+c\right)^2-4\,b\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*c
os(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*
```

```
cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2*b*c
os(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4*b)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx)) \csc^5 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**5, x)
```

Giac [A]
time = 0.54, size = 177, normalized size = 1.81

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) - 120*b*
tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*x +
1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 3.85, size = 211, normalized size = 2.15

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{4\,b^2}{\frac{3\,a\,b}{2}-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{3\,a\,b}{2}-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a}{4}}{16\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) - (2*b*atan
h((4*b^2)/((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)) - (3*a*b*tan(c/2 + (d*x)/2
))/(2*((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)))))/d + (a*tan(c/2 + (d*x)/2)^4
)/(64*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2)))/
(8*d) - (a/4 + (2*b*tan(c/2 + (d*x)/2))/3 + 2*a*tan(c/2 + (d*x)/2)^2 + 10*b
*tan(c/2 + (d*x)/2)^3)/(16*d*tan(c/2 + (d*x)/2)^4)
```

3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-b*cot(d*x+c)^2/d-2/3*a*cot(d*x+c)^3/d-1/4*b*cot(d*x+c)^4/d-
1/5*a*cot(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {780}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) -
 (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]]
)/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)^2}{x^6}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^6}+\frac{b}{x^5}+\frac{2a}{x^4}+\frac{2b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 104, normalized size = 1.20

$$-\frac{8a\cot(c+dx)}{15d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{b(2\csc^2(c+dx)+\csc^4(c+dx)+4\log(\cos(c+dx))-4\log(\sin(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*(2*Csc[c + d*x]^2 + Csc[c + d*x]^4
+ 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4*d)
```

Maple [A]
time = 0.26, size = 66, normalized size = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^{4}(dx+c)\right)}{5}-\frac{4\left(\csc^{2}(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{1}{4\sin(dx+c)^{4}}-\frac{1}{2\sin(dx+c)^{2}}+\ln(\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^{4}(dx+c)\right)}{5}-\frac{4\left(\csc^{2}(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{1}{4\sin(dx+c)^{4}}-\frac{1}{2\sin(dx+c)^{2}}+\ln(\tan(dx+c))\right)}{d}$
risch	$\frac{2b\,e^{8i(dx+c)}-10b\,e^{6i(dx+c)}-\frac{32ia\,e^{4i(dx+c)}}{3}+10b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}-2b\,e^{2i(dx+c)}-\frac{16ia}{15}}{d\left(e^{2i(dx+c)}-1\right)^{5}}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/4/sin(d*
x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c))))
```

Maxima [A]
time = 0.34, size = 72, normalized size = 0.83

$$\frac{60\,b\log\left(\tan\left(dx+c\right)\right)-\frac{60\,a\tan(dx+c)^{4}+60\,b\tan(dx+c)^{3}+40\,a\tan(dx+c)^{2}+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 +
 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(81) = 162$.
time = 0.34, size = 174, normalized size = 2.00

$$-\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+30\,\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-30\,\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^{2}+\frac{1}{4}\right)\sin\left(dx+c\right)+60\,a\cos\left(dx+c\right)-15\,\left(2\,b\cos\left(dx+c\right)^{2}-3\,b\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x + c
)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) +
 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))\csc^6(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**6, x)
```

Giac [A]
time = 0.50, size = 84, normalized size = 0.97

$$\frac{60\,b\log\left(\left|\tan\left(dx+c\right)\right|\right) - \frac{137\,b\tan(dx+c)^5+60\,a\tan(dx+c)^4+60\,b\tan(dx+c)^3+40\,a\tan(dx+c)^2+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x +
c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a
)/tan(d*x + c)^5)/d
```

Mupad [B]
time = 3.93, size = 70, normalized size = 0.80

$$\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{a\tan(c+d\,x)^4 + b\tan(c+d\,x)^3 + \frac{2\,a\tan(c+d\,x)^2}{3} + \frac{b\tan(c+d\,x)}{4} + \frac{a}{5}}{d\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/5 + (b*tan(c + d*x))/4 + (2*a*tan(c + d*x)^2)/
3 + a*tan(c + d*x)^4 + b*tan(c + d*x)^3)/(d*tan(c + d*x)^5)
```

3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=113

$$\frac{3}{8}\left(a^2-5b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}$$

```
[Out] 3/8*(a^2-5*b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+1/8*cos(d*x+c)^2*
(7*b-5*a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*ta
n(d*x+c))^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1659, 1824, 649, 209, 266}

$$\frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^2}{4d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
+ (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (C
os[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
```

```
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^2}{(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} - \frac{\text{Subst}\left(\int \frac{(a+x)(ab^4+ \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+ \cdots}{\cdots}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+ \cdots}{\cdots}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+ \cdots}{8d}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+ \cdots}{8d}$$

$$= \frac{3}{8}\left(a^2-5b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+ \cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(113) = 226.

time = 3.77, size = 240, normalized size = 2.12

$$\frac{b\left(\frac{4(-2a^2+3b^2)\text{ArcTan}(\tan(c+dx))}{b}+16a\cos^2(c+dx)-4a\cos^4(c+dx)+4\left(2a+\frac{a^2-3b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+4\left(2a+\frac{-a^2+3b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+\frac{2(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{b}+\frac{2(-2a^2+3b^2)\sin(2(c+dx))}{b}+\frac{3(a^2-b^2)(2\text{ArcTan}(\tan(c+dx))+\sin(2(c+dx)))}{2b}+8b\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(-2*a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a
*Cos[c + d*x]^4 + 4*(2*a + (a^2 - 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan
[c + d*x]] + 4*(2*a + (-a^2 + 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + (2*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(-2*a^2 + 3*b^2
)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c +
 d*x)]))/(2*b) + 8*b*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.15, size = 140, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(dx}{\cos(dx}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(dx}{\cos(dx}\right.}{d}$
risch	$\frac{2ib^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{3a^2x}{8}-\frac{15b^2x}{8}+\frac{3\,e^{2i(dx+c)}ab}{8d}-\frac{ie^{2i(dx+c)}b^2}{4d}-\frac{ie^{-2i(dx+c)}a^2}{8d}+\frac{3\,e^{-2i(dx+c)}ab}{8d}+\frac{4iabc}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*
b*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^7/cos
(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x
-15/8*c))
```

Maxima [A]
time = 0.55, size = 128, normalized size = 1.13

$$\frac{8\,ab\log\left(\tan\left(dx+c\right)^2+1\right)+8\,b^2\tan\left(dx+c\right)+3\,(a^2-5\,b^2)(dx+c)+\frac{16\,ab\tan(dx+c)^2-(5\,a^2-9\,b^2)\tan(dx+c)^3+12\,ab-(3\,a^2-7\,b^2)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a*b*log(tan(d*x + c)^2 + 1) + 8*b^2*tan(d*x + c) + 3*(a^2 - 5*b^2)*(
d*x + c) + (16*a*b*tan(d*x + c)^2 - (5*a^2 - 9*b^2)*tan(d*x + c)^3 + 12*a*b
 - (3*a^2 - 7*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 137, normalized size = 1.21

$$-\frac{8\,ab\cos\left(dx+c\right)^5-32\,ab\cos\left(dx+c\right)^3+32\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-\left(6\,(a^2-5\,b^2)dx-13\,ab\right)\cos\left(dx+c\right)-2\left(2\,(a^2-b^2)\cos\left(dx+c\right)^4-(5\,a^2-9\,b^2)\cos\left(dx+c\right)^2+8\,b^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cos(d*x + c)^5 - 32*a*b*cos(d*x + c)^3 + 32*a*b*cos(d*x + c)*l
og(-cos(d*x + c)) - (6*(a^2 - 5*b^2)*d*x - 13*a*b)*cos(d*x + c) - 2*(2*(a^2
 - b^2)*cos(d*x + c)^4 - (5*a^2 - 9*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5784 vs. 2(107) = 214.
time = 2.75, size = 5784, normalized size = 51.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 + 24*a^2*d*x
*tan(d*x)^5*tan(c)^5 - 120*b^2*d*x*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*t
an(c)^5 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 3*pi*b
^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^3*tan(c)^5 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(
tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 64*a*b*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 48*a^2*d*x*tan(d*x)^5*tan(c
)^3 - 240*b^2*d*x*tan(d*x)^5*tan(c)^3 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 24*a^2*
d*x*tan(d*x)^4*tan(c)^4 + 120*b^2*d*x*tan(d*x)^4*tan(c)^4 - 3*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
```

```
4*tan(c)^4 + 48*a^2*d*x*tan(d*x)^3*tan(c)^5 - 240*b^2*d*x*tan(d*x)^3*tan(c)
^5 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^3*tan(c)^5 + 44*a*b*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(
2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c) - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c
)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^4*tan(c)^2 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
3*tan(c)^3 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d
*x)^5*tan(c)^3 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(d*x)^5*tan(c)^3 - 128*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)^5*tan(c)^3 - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*
tan(c)^4 - 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)
^4*tan(c)^4 + 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(
d*x)^4*tan(c)^4 + 64*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)^4*tan(c)^4 + 24*a^2*tan(d*x)^5*tan(c)^4 - 120*b^2*tan(d*x)^5*tan(c
)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 12*b^2*arcta
n((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^5 - 12*b^2*a
rctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 - 128
*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^5
+ 24*a^2*tan(d*x)^4*tan(c)^5 - 120*b^2*tan(d*x)^4*tan(c)^5 + 24*a^2*d*x*tan
(d*x)^5*tan(c) - 120*b^2*d*x*tan(d*x)^5*tan(c) + 3*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c) -
48*a^2*d*x*tan(d*x)^4*tan(c)^2 + 240*b^2*d*x*tan(d*x)^4*tan(c)^2 - 6*pi*b^2
*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*ta
n(d*x)^4*tan(c)^2 + 96*a^2*d*x*tan(d*x)^3*tan(c)^3 - 480*b^2*d*x*tan(d*x)^3
*tan(c)^3 + 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*ta
n(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 24*a*b*tan(d*x)^5*tan(c)^3 - 48*a^
2*d*x*tan(d*x)^2*tan(c)^4 + 240*b^2*d*x*tan(d*x)^2*tan(c)^4 - 6*pi*b^2*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x
)^2*tan(c)^4 - 172*a*b*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x*tan(d*x)*tan(c)^5 -
 120*b^2*d*x*tan(d*x)*tan(c)^5 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 24*a*b*tan(d*x)^
3*tan(c)^5 - 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*pi*b^2*sgn
(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
+ 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c) + 6*b^2*arctan((tan(d*x) + tan(c
))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c) - 6*b^2*arctan(-(tan(d*x) - tan
(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c) - 64*a*b*log(4*(tan(d*x)^4*ta
n(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x
```

```
)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c) - 12*pi*b^2*sgn(2*tan(d*x)^
2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)^2*tan(c)^2 - 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d
*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 + 12*b^2*a...
```

Mupad [B]
time = 3.82, size = 127, normalized size = 1.12

$$x\left(\frac{3a^2}{8}-\frac{15b^2}{8}\right)+\frac{b^2\tan(c+dx)}{d}+\frac{\left(\frac{9b^2}{8}-\frac{5a^2}{8}\right)\tan(c+dx)^3+2ab\tan(c+dx)^2+\left(\frac{7b^2}{8}-\frac{3a^2}{8}\right)\tan(c+dx)+\frac{3ab}{2}}{d\left(\tan(c+dx)^4+2\tan(c+dx)^2+1\right)}+\frac{ab\ln\left(\tan(c+dx)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] x*((3*a^2)/8 - (15*b^2)/8) + (b^2*tan(c + d*x))/d + ((3*a*b)/2 - tan(c + d*
x)*((3*a^2)/8 - (7*b^2)/8) - tan(c + d*x)^3*((5*a^2)/8 - (9*b^2)/8) + 2*a*b
*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) + (a*b*log(tan
(c + d*x)^2 + 1))/d
```

3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+2*b^2*cos(d*x+c)/d+1/3*a^2*cos
(d*x+c)^3/d-1/3*b^2*cos(d*x+c)^3/d+b^2*sec(d*x+c)/d-2*a*b*sin(d*x+c)/d-2/3*
a*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{2b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x
])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c
 + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \left(a^2\sin^3(c+dx)+2ab\sin^3(c+dx)\tan(c+dx)+b^2\sin^3(c+dx\right.\\ &= a^2\int \sin^3(c+dx)\,dx+(2ab)\int \sin^3(c+dx)\tan(c+dx)\,dx+b^2\int\\ &= -\frac{a^2\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{(2ab)\text{Subst}\left(\int\frac{x^4}{1-x^2}\,dx\right.}{d}\\ &= -\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{(2ab)\text{Subst}\left(\int\left(-1-x^2+\frac{}{1-}\right.\right.}{d}\\ &= -\frac{a^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{b^2\cos^3(c-}{3d}\\ &= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d}+\frac{a^2}{}\end{aligned}$$

Mathematica [A]
time = 1.01, size = 152, normalized size = 1.25

$$\frac{\sec(c+dx)\left(-9a^2+45b^2+\left(-8a^2+20b^2\right)\cos(2(c+dx))+\left(a^2-b^2\right)\cos(4(c+dx))-48ab\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+48ab\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-28ab\sin(2(c+dx))+2ab\sin(4(c+dx))\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2
- b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

Maple [A]
time = 0.16, size = 113, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4\right.\right.}{d}$
default	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4\right.\right.}{d}$
risch	$\frac{5i e^{i(dx+c)}ab}{4d}-\frac{3\,e^{i(dx+c)}a^2}{8d}+\frac{7\,e^{i(dx+c)}b^2}{8d}-\frac{5i e^{-i(dx+c)}ab}{4d}-\frac{3\,e^{-i(dx+c)}a^2}{8d}+\frac{7\,e^{-i(dx+c)}b^2}{8d}+\frac{2b^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/3*sin(d*x+c)^3-sin(d*x+
c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^
4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.41, size = 104, normalized size = 0.85

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{2}-\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)ab-\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b^{2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a*b - (cos(d*x +
c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2)/d
```

Fricas [A]
time = 0.35, size = 126, normalized size = 1.03

$$\frac{\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-3\,\left(a^{2}-2\,b^{2}\right)\cos\left(dx+c\right)^{2}+3\,b^{2}+2\,\left(ab\cos\left(dx+c\right)^{3}-4\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - b^2)*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 3*(a^2 - 2*b^2)*cos(d*x + c)^
2 + 3*b^2 + 2*(a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 34422 vs. 2(116) = 232.
time = 15.30, size = 34422, normalized size = 282.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^8*tan(1/2*c)^8 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^8*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 30*pi*b^2
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
```

```
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c
)^7 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 + 30*pi*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 30*pi*b^2
*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30*b^2*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30*b^2*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 -
192*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^8*tan(1/2*c)^8 + 192*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 30*pi*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 60*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 60*pi*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 - 60*pi*b^2*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 + 120*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 - 30*pi*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
```

```
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c...
```

Mupad [B]
time = 6.71, size = 174, normalized size = 1.43

$$\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{8\,a^2}{3}-\frac{32\,b^2}{3}\right)-4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a^2}{3}-\frac{16\,b^2}{3}+\frac{28\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{28\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^2*((8*a^2)/3 - (3
2*b^2)/3) - 4*a^2*tan(c/2 + (d*x)/2)^4 + (4*a^2)/3 - (16*b^2)/3 + (28*a*b*t
an(c/2 + (d*x)/2)^3)/3 - (28*a*b*tan(c/2 + (d*x)/2)^5)/3 - 4*a*b*tan(c/2 +
(d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c
/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1))
```

3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=76

$$\frac{1}{2}\left(a^2-3b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 1/2*(a^2-3*b^2)*x-2*a*b*ln(cos(d*x+c))/d+3/2*b^2*tan(d*x+c)/d-1/2*cos(d*x+c
)*sin(d*x+c)*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1659, 788, 649, 209, 266}

$$\frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 3*b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^2}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)(-ab}{b^2+}\right.}{}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} -$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} +$$

$$= \frac{1}{2}(a^2-3b^2)\,x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(76) = 152.

time = 2.67, size = 162, normalized size = 2.13

$$\frac{b\left(\frac{(-a^2+b^2)\text{ArcTan}(\tan(c+dx))}{b} + 2a\cos^2(c+dx) + \left(2a+\frac{a^2-2b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + \left(2a+\frac{-a^2+2b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right) + \frac{(-a^2+b^2)\sin(2(c+dx))}{2b} + 2b\tan(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

[Out] (b*(((-a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a^2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (-a^2 + 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((-a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)

Maple [A]
time = 0.16, size = 109, normalized size = 1.43

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\text{co}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\text{co}\right.}{d}$
risch	$2iabx+\frac{a^2x}{2}-\frac{3b^2x}{2}+\frac{e^{2i(dx+c)}ab}{4d}+\frac{ie^{2i(dx+c)}a^2}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{e^{-2i(dx+c)}ab}{4d}-\frac{ie^{-2i(dx+c)}a^2}{8d}+\frac{ie^{-2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))

Maxima [A]
time = 0.58, size = 82, normalized size = 1.08

$$\frac{2\,ab\log\left(\tan\left(dx+c\right)^2+1\right)+2\,b^2\tan\left(dx+c\right)+(a^2-3\,b^2)(dx+c)+\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d*x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d

Fricas [A]
time = 0.34, size = 101, normalized size = 1.33

$$\frac{2\,ab\cos\left(dx+c\right)^3-4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left((a^2-3\,b^2)dx-ab\right)\cos\left(dx+c\right)-\left((a^2-b^2)\cos\left(dx+c\right)^2-2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

```
[Out] 1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2 -
 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*sin(
d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^2 \sin^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. 2(70) = 140.
time = 0.72, size = 1061, normalized size = 13.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + a^2*d
*x*tan(d*x)^3*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan
(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d
*x*tan(d*x)*tan(c)^3 + a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(d*x)^4*ta
n(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x
)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c) + 2*a*b*log(4*(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + a^2*tan(d*x)^3*tan(c)^
2 - 3*b^2*tan(d*x)^3*tan(c)^2 - 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(c)^2 + 1))*tan(d*x)*tan(c)^3 + a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2
*tan(c)^3 - a^2*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(d*x)*ta
n(c) - 3*b^2*d*x*tan(d*x)*tan(c) - a*b*tan(d*x)^3*tan(c) - a^2*d*x*tan(c)^2
 + 3*b^2*d*x*tan(c)^2 - 5*a*b*tan(d*x)^2*tan(c)^2 - a*b*tan(d*x)*tan(c)^3 +
 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 2*b^2
*tan(d*x)^3 - 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d
*x)*tan(c) - 2*a^2*tan(d*x)^2*tan(c) + 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
```

```
 1)/(tan(c)^2 + 1))*tan(c)^2 - 2*a^2*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a
^2*d*x + 3*b^2*d*x + a*b*tan(d*x)^2 + 5*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2
+ 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + a^2*tan(d*x) -
3*b^2*tan(d*x) + a^2*tan(c) - 3*b^2*tan(c) - a*b)/(d*tan(d*x)^3*tan(c)^3 +
d*tan(d*x)^3*tan(c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d
*x)^2 + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

Mupad [B]
time = 3.69, size = 75, normalized size = 0.99

$$\frac{\cos\left(c+d\,x\right)^2\,\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)+b^2\,\tan(c+d\,x)+a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)+d\,x\,\left(\frac{a^2}{2}-\frac{3\,b^2}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)) + b^2*tan(c + d*x) + a
*b*log(tan(c + d*x)^2 + 1) + d*x*(a^2/2 - (3*b^2)/2))/d
```

3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+b^2*cos(d*x+c)/d+b^2*sec(d*x+c
)/d-2*a*b*sin(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {3598, 2718, 2672, 327, 212, 2670, 14}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])
/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \left(a^2\sin(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\sin(c+dx)\tan^{\ldots}\right.\\ &= a^2\int \sin(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \sin\ldots\\ &= -\frac{a^2\cos(c+dx)}{d}+\frac{(2ab)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\frac{b^2\text{Subst}\ldots}{\ldots}\\ &= -\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{(2ab)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c\ldots\right.}{d}\\ &= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(\ldots}{\ldots}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 111, normalized size = 1.63

$$-\frac{\sec(c+dx)\left(a^2-3b^2+\left(a^2-b^2\right)\cos(2(c+dx))+4ab\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2ab\sin(2(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

[Out] -1/2*(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)]))/d

Maple [A]
time = 0.16, size = 83, normalized size = 1.22

method	result
derivativedivides	$\frac{-a^2\cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{-a^2\cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
risch	$\frac{i e^{i(dx+c)}ab}{d}-\frac{e^{i(dx+c)}a^2}{2d}+\frac{e^{i(dx+c)}b^2}{2d}-\frac{i e^{-i(dx+c)}ab}{d}-\frac{e^{-i(dx+c)}a^2}{2d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{2b^2 e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+2$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^2*cos(d*x+c)+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))

Maxima [A]
time = 0.36, size = 67, normalized size = 0.99

$$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-a^2\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] (b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d

Fricas [A]
time = 0.34, size = 90, normalized size = 1.32

$$\frac{ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)-2\,ab\cos(dx+c)\sin(dx+c)-(a^2-b^2)\cos(dx+c)^2+b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b^2)/(d*cos(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2837 vs. $2(68) = 136$.
time = 1.10, size = 2837, normalized size = 41.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 -
 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 4*a*b*log(2*(tan
(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c
)^3 - 4*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4
 - 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 +
8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - a*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
```

```
4 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^4 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(
1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*tan(1/2*d*x)^4*tan
(1/2*c) + 24*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 4*a*b*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan
(1/2*c)^3 + 24*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - a*b*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + a*b*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 4*a*b*tan(1/2*d*
x)*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4 - 2*b^2*tan(1/2*d*x)^4 + 8*a^2*tan(1/2
*d*x)^3*tan(1/2*c) - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 20*a^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 24*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*tan(1/2*d*x)*ta
n(1/2*c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + a^2*tan(1/2*c)^4 - 2*b^2*tan
(1/2*c)^4 - 4*a*b*tan(1/2*d*x)^3 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1...
```

Mupad [B]

time = 4.04, size = 93, normalized size = 1.37

$$\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2 +
 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^4 - 1))
```

3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3598, 3855, 2686, 8}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*S
ec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc(c+dx)+2ab\sec(c+dx)+b^2\sec(c+dx)\tan(c+dx)\right)\,dx$$
$$= a^2\int \csc(c+dx)\,dx + (2ab)\int \sec(c+dx)\,dx + b^2\int \sec(c+dx)\tan$$
$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2\text{Subst}(\int 1$$
$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2\sec(c+dx}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.

time = 0.27, size = 97, normalized size = 2.26

$$\frac{a\left(-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + b^2*Sec[c + d*x])/d
```

Maple [A]

time = 0.23, size = 56, normalized size = 1.30

method	result	size
derivativedivides	$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))+2ab\ln(\sec(dx+c)+\tan(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$	56
default	$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))+2ab\ln(\sec(dx+c)+\tan(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$	56
risch	$\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{i(dx+c)}+i)}{d}-\frac{2ab\ln(e^{i(dx+c)}-i)}{d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(csc(d*x+c)-cot(d*x+c))+2*a*b*ln(sec(d*x+c)+tan(d*x+c))+b^2/cos(
d*x+c))
```

Maxima [A]

time = 0.33, size = 60, normalized size = 1.40

$$\frac{ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-a^2\log(\cot(dx+c)+\csc(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c)
 + csc(d*x + c)) + b^2/cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(43) = 86$.
time = 0.35, size = 102, normalized size = 2.37

$$-\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2ab\cos(dx+c)\log(\sin(dx+c)+1)+2ab\cos(dx+c)\log(-\sin(dx+c)+1)-2b^2}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*b*
cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)
```

Giac [A]
time = 0.57, size = 74, normalized size = 1.72

$$\frac{2ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-2ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{2b^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*b^2/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

Mupad [B]
time = 3.72, size = 125, normalized size = 2.91

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b^2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{16\,a^2\,b^2}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(c + d*x))^2/sin(c + d*x),x)

[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (2*b^2)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) - (4*a*b*atanh((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (8*a^3*b*tan(c/2 + (d*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d

3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 45}

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*Cot[c + d*x])/d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/
d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)\,dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.
time = 0.63, size = 91, normalized size = 2.17

$$-\frac{\cos(c+dx)\,(a\cos(c+dx)(a\cot(c+dx)+2b(\log(\cos(c+dx))-\log(\sin(c+dx))))-b^2\sin(c+dx))\,(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] -
Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2))
```

Maple [A]
time = 0.23, size = 38, normalized size = 0.90

method	result	size
derivativedivides	$\frac{-a^2\cot(dx+c)+2ab\ln(\tan(dx+c))+b^2\tan(dx+c)}{d}$	38
default	$\frac{-a^2\cot(dx+c)+2ab\ln(\tan(dx+c))+b^2\tan(dx+c)}{d}$	38
risch	$-\frac{2i\left(a^2 e^{2i(dx+c)}-b^2 e^{2i(dx+c)}+a^2+b^2\right)}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}+\frac{2ab\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{2ab\ln\left(e^{2i(dx+c)}+1\right)}{d}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cot(d*x+c)+2*a*b*ln(tan(d*x+c))+b^2*tan(d*x+c))
```

Maxima [A]
time = 0.33, size = 39, normalized size = 0.93

$$\frac{2\,ab\log\left(\tan\left(dx+c\right)\right)+b^2\tan\left(dx+c\right)-\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.
time = 0.34, size = 96, normalized size = 2.29

$$-\frac{ab\cos\left(dx+c\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+\left(a^2+b^2\right)\cos\left(dx+c\right)^2-b^2}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log(
-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^2)
/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \csc^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)
```

Giac [**A**]
time = 0.57, size = 51, normalized size = 1.21

$$\frac{2\,ab \log (|\tan (dx + c)|) + b^2 \tan (dx + c) - \frac{2\,ab \tan(dx+c)+a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a^
2)/tan(d*x + c))/d
```

Mupad [**B**]
time = 3.64, size = 44, normalized size = 1.05

$$\frac{b^2 \tan(c + d\,x)}{d} - \frac{a^2}{d \tan (c + d\,x)} + \frac{2\,a\,b\,\ln (\tan(c + d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d - a^2/(d*tan(c + d*x)) + (2*a*b*log(tan(c + d*x)))/d
```

3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)}{2}$$

```
[Out] -1/2*a^2*arctanh(cos(d*x+c))/d-b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(
d*x+c))/d-2*a*b*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+b^2*sec(d*x+c)
/d
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3598, 3853, 3855, 2701, 327, 213, 2702}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*ArcTanh[Cos[c + d*x]])/d - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*b
*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[
c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^3(c+dx)+2ab\csc^2(c+dx)\sec(c+dx)+b^2\csc(c+dx)\right.$$

$$= a^2\int \csc^3(c+dx)\,dx+(2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2\int \csc(c+dx)\,dx-\frac{(2ab)\text{Subst}}{}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c}{2d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(95) = 190.

time = 1.99, size = 250, normalized size = 2.63

$$\frac{8b^2-8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4(a^2+2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{8b^2\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{8b^2\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}-8ab\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*
Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A]
time = 0.24, size = 101, normalized size = 1.06

method	result
derivativedivides	$\frac{a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\cos(dx+c)}+\ln(\mathrm{cs}\right.}{d}$
default	$\frac{a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\cos(dx+c)}+\ln(\mathrm{cs}\right.}{d}$
risch	$\frac{a^2 e^{5i(dx+c)}+2b^2 e^{5i(dx+c)}-4iab\, e^{5i(dx+c)}+2a^2 e^{3i(dx+c)}-4b^2 e^{3i(dx+c)}+a^2 e^{i(dx+c)}+2b^2 e^{i(dx+c)}+4iab\, e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)} - 2ab\,\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(
-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-co
t(d*x+c))))
```

Maxima [A]
time = 0.33, size = 122, normalized size = 1.28

$$\frac{a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+2\,b^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-4\,ab\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 4*a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(91) = 182.
time = 0.41, size = 230, normalized size = 2.42

$$\frac{8\,ab\cos(dx+c)\sin(dx+c)+2\,(a^2+2\,b^2)\cos(dx+c)^2-4\,b^2-\left((a^2+2\,b^2)\cos(dx+c)^3-(a^2+2\,b^2)\cos(dx+c)\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+\left((a^2+2\,b^2)\cos(dx+c)^3-(a^2+2\,b^2)\cos(dx+c)\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+4\,\left(ab\cos(dx+c)^3-ab\cos(dx+c)\right)\log(\sin(dx+c)+1)-4\,\left(ab\cos(dx+c)^3-ab\cos(dx+c)\right)\log(-\sin(dx+c)+1)}{4\,\left(d\cos(dx+c)^3-d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(d*x + c)*sin(d*x + c) + 2*(a^2 + 2*b^2)*cos(d*x + c)^2 - 4*b
^2 - ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c
))*log(sin(d*x + c) + 1) - 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-s
in(d*x + c) + 1))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \csc^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)
```

Giac [A]

time = 0.59, size = 172, normalized size = 1.81

$$\frac{a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 16\,ab \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 16\,ab \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - 8\,ab \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 4\,(a^2 + 2\,b^2) \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{16\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \frac{6\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c) +
 4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2
+ 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]

time = 3.84, size = 292, normalized size = 3.07

$$\frac{a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2}{8\,d} + \frac{\ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{2} + b^2\right)}{d} + \frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2} + 8\,b^2\right) - \frac{a^2}{2} + 4\,a\,b \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 - 4\,a\,b \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\,\left(4 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 4 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{8\,a\,b^3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b-16 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3} - \frac{16\,a^2\,b^2}{4\,a^3\,b-16 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3} + \frac{4\,a^3\,b \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b-16 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3}\right)}{d} - \frac{a\,b \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(a^2/2 + b^2))/
d + (tan(c/2 + (d*x)/2)^2*(a^2/2 + 8*b^2) - a^2/2 + 4*a*b*tan(c/2 + (d*x)/2
)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^4)) + (4*a*b*atanh((8*a*b^3*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(8*a*b^3 + 4*a^3*b - 16*a^2*
b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (a*b*tan(c/2 + (d*x)/2))/d
```

3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=79

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/d-a*b*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d+2*a*b*ln(
tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x
]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)}{x^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2b^2}{x^4}+\frac{2ab^2}{x^3}+\frac{a^2+b^2}{x^2}+\frac{2a}{x}\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{2ab\,\text{l}\end{aligned}$$

Mathematica [A]
time = 1.51, size = 127, normalized size = 1.61

$$-\frac{\left(3ab\cot^2(c+dx)+a^2\cot^3(c+dx)+\cos^2(c+dx)\left((2a^2+3b^2)\cot(c+dx)+6ab(\log(\cos(c+dx))-\log(\sin(c+dx)))\right)-\frac{3}{2}b^2\sin(2(c+dx))\right)(a+b\tan(c+dx))^2}{3d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/3*((3*a*b*Cot[c + d*x]^2 + a^2*Cot[c + d*x]^3 + Cos[c + d*x]^2*((2*a^2 +
 3*b^2)*Cot[c + d*x] + 6*a*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - (3*
b^2*Sin[2*(c + d*x)])/2)*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)
```

Maple [A]
time = 0.21, size = 80, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
risch	$\frac{4ab\,e^{6i(dx+c)}+4ia^2e^{4i(dx+c)}-4ib^2e^{4i(dx+c)}+\frac{8ia^2e^{2i(dx+c)}}{3}+8ib^2e^{2i(dx+c)}-4ab\,e^{2i(dx+c)}-\frac{4ia^2}{3}-4ib^2}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^3}+\frac{2ab\ln\left(e^{2i(dx\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/2/sin(d*x+c)^2+ln(tan
(d*x+c)))+b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))
```

Maxima [A]
time = 0.35, size = 69, normalized size = 0.87

$$\frac{6\,ab\log\left(\tan\left(dx+c\right)\right)+3\,b^2\tan\left(dx+c\right)-\frac{3\,ab\tan(dx+c)+3\,(a^2+b^2)\tan(dx+c)^2+a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*log(tan(d*x + c)) + 3*b^2*tan(d*x + c) - (3*a*b*tan(d*x + c) + 3
*(a^2 + b^2)*tan(d*x + c)^2 + a^2)/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.
time = 0.36, size = 174, normalized size = 2.20

$$-\frac{2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^4-3\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2+3\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-3\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+3\,b^2}{3\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 3*b^2)*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)*sin(d*x + c) - 3*
(a^2 + 3*b^2)*cos(d*x + c)^2 + 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*lo
g(cos(d*x + c)^2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*
log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3 - d
*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^2 \csc^4{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**4, x)
```

Giac [A]
time = 0.59, size = 91, normalized size = 1.15

$$\frac{6\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+3\,b^2\tan\left(dx+c\right)-\frac{11\,ab\tan\left(dx+c\right)^3+3\,a^2\tan\left(dx+c\right)^2+3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x +
c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a
^2)/tan(d*x + c)^3)/d
```

Mupad [B]
time = 3.79, size = 72, normalized size = 0.91

$$\frac{b^2\tan(c+d\,x)}{d}-\frac{\tan(c+d\,x)^2\,\left(a^2+b^2\right)+\frac{a^2}{3}+a\,b\tan(c+d\,x)}{d\tan\left(c+d\,x\right)^3}+\frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^2*(a^2 + b^2) + a^2/3 + a*b*tan(c + d*
x))/(d*tan(c + d*x)^3) + (2*a*b*log(tan(c + d*x)))/d
```

3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3b^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{3a^2\cot(c$$

```
[Out] -3/8*a^2*arctanh(cos(d*x+c))/d-3/2*b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(
sin(d*x+c))/d-2*a*b*csc(d*x+c)/d-3/8*a^2*cot(d*x+c)*csc(d*x+c)/d-2/3*a*b*cs
c(d*x+c)^3/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+3/2*b^2*sec(d*x+c)/d-1/2*b^2
*csc(d*x+c)^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^2\sec(c+dx)}{2d}-\frac{3b^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
 d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^5(c+dx) + 2ab\csc^4(c+dx)\sec(c+dx) + b^2\csc^3(c+dx\right.$$
$$= a^2\int \csc^5(c+dx)\,dx + (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + b^2\int$$
$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a^2)\int \csc^3(c+dx)\,dx - \frac{(2ab)}{}$$
$$= -\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{b^2\,c}{}$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2ab\csc(c+dx)}{d} - \frac{3a^2\cot(c+dx)\,\mathrm{cs}}{8d}$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3b^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2ab\tanh^{}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 994 vs. 2(165) = 330.
time = 6.21, size = 994, normalized size = 6.02

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2) - (7*a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + ((-3*a^2 - 4*b^2)*Cos[c + d*x]^2
*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2) - (a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a +
b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (a^2*Cos[c
+ d*x]^2*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2) - (3*(a^2 + 4*b^2)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]]
*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (2*a*b
*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (3*(a^2 + 4*b^2)*Cos[c + d*
x]^2*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2) + (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (
(3*a^2 + 4*b^2)*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(
32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (a^2*Cos[c + d*x]^2*Sec[(c + d*
x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
+ (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c +
```

```
d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^2*Co
s[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (7*a*b*Cos[c + d
*x]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(6*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2) - (a*b*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(a
+ b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.28, size = 145, normalized size = 0.88

method	result
derivativedivides	$\frac{a^{2}\left(\left(-\frac{\left(\csc^{3}(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^{3}}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c\right.}{d}$
default	$\frac{a^{2}\left(\left(-\frac{\left(\csc^{3}(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^{3}}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c\right.}{d}$
risch	$\frac{9a^{2}e^{9i(dx+c)}+36b^{2}e^{9i(dx+c)}-48iab\,e^{9i(dx+c)}-24a^{2}e^{7i(dx+c)}-96b^{2}e^{7i(dx+c)}+160iab\,e^{7i(dx+c)}-66a^{2}e^{5i(dx+c)}+120b^{2}e^{5i}}{12d\left(e^{2i(dx+c)}-1\right)^{4}\left(e^{2i(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-c
ot(d*x+c)))+2*a*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))
)+b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d*
x+c))))
```

Maxima [A]
time = 0.36, size = 187, normalized size = 1.13

$$\frac{3\,a^{2}\left(\frac{2\left(3\,\cos(dx+c)^{3}-5\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,b^{2}\left(\frac{2\left(3\,\cos(dx+c)^{2}-2\right)}{\cos(dx+c)^{3}-\cos(dx+c)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-16\,ab\left(\frac{2\left(3\,\sin(dx+c)^{2}+1\right)}{\sin(dx+c)^{3}}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 16*a*b*(2*(3*sin(d*x + c)^2 + 1)/s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(151) = 302$.
time = 0.39, size = 333, normalized size = 2.02

$$\frac{18\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{4}-30\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{2}+48\,b^{2}-9\left(\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{5}-2\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{3}+\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+9\left(\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{5}-2\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{3}+\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+48\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-48\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+32\left(3\,ab\cos\left(dx+c\right)^{3}-4\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^{5}-2\,d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(18*(a^2 + 4*b^2)*cos(d*x + c)^4 - 30*(a^2 + 4*b^2)*cos(d*x + c)^2 + 4
8*b^2 - 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 +
(a^2 + 4*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 + 4*b^2)*
cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x + c
))*log(-1/2*cos(d*x + c) + 1/2) + 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1) - 48*(a*b*cos(d*x + c)^5 - 2
*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1) + 32*(3*a*b*
cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5 - 2*d*
cos(d*x + c)^3 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2\csc^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**5, x)
```

Giac [A]
time = 0.63, size = 269, normalized size = 1.63

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-384\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,(a^2+4\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{384\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+600\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*a*b*log(abs(
tan(1/2*d*x + 1/2*c) + 1)) - 384*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
40*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
)) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*tan(1/2*d*x + 1/2*c)^4
 + 600*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^2
*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*
x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 3.91, size = 378, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{8}+\frac{3\,b^2}{2}\right)}{d}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{7\,a^2}{4}+2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(2\,a^2+34\,b^2)+\frac{a^2}{4}+\frac{56\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-20\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{8}+\frac{b^2}{8}\right)}{d}-\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{12\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^3\,b-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}-\frac{16\,a^2\,b^2}{3\,a^3\,b-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}+\frac{3\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^3\,b-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}\right)}{d}-\frac{5\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((3*a^2)/8 + (3*b^2)/2))/d + (a^2*tan(c/2 + (d*x)/
2)^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((7*a^2)/4 + 2*b^2) - tan(c/2 + (d*x)/
2)^4*(2*a^2 + 34*b^2) + a^2/4 + (56*a*b*tan(c/2 + (d*x)/2)^3)/3 - 20*a*b*ta
n(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2
)^4 - 16*tan(c/2 + (d*x)/2)^6)) + (tan(c/2 + (d*x)/2)^2*(a^2/8 + b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (4*a*b*atanh((12*a*b^3*tan(c/2 + (d*x
)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(
12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) + (3*a^3*b*tan(c/2 + (d
*x)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (5*a*b*t
an(c/2 + (d*x)/2))/(4*d)
```

3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\cot^2(c+dx)}{d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{2a$$

```
[Out] -(a^2+2*b^2)*cot(d*x+c)/d-2*a*b*cot(d*x+c)^2/d-1/3*(2*a^2+b^2)*cot(d*x+c)^3
/d-1/2*a*b*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*a*b*ln(tan(d*x+c))/d+b^2
*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 962}

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^
2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5
)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{2a^2b^2+b^4}{x^4}+\frac{4ab^2}{x^3}+\frac{a^2+2b^2}{x^2}+\frac{2a}{x}\right)\,dx, x, b\right.}{d}$$

$$= -\frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{2ab\cot^2(c+dx)}{d} - \frac{(2a^2+b^2)\cot^3(c+d}{3d}$$

Mathematica [A]
time = 1.63, size = 114, normalized size = 0.93

$$-\frac{2\cot(c+dx)\left(8a^2+25b^2+(4a^2+5b^2)\csc^2(c+dx)+3a^2\csc^4(c+dx)\right)+15b\left(2a\csc^2(c+dx)+a\csc^4(c+dx)+4a\log(\cos(c+dx))-4a\log(\sin(c+dx))-2b\tan(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/30*(2*Cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3*
a^2*Csc[c + d*x]^4) + 15*b*(2*a*Csc[c + d*x]^2 + a*Csc[c + d*x]^4 + 4*a*Log
[Cos[c + d*x]] - 4*a*Log[Sin[c + d*x]] - 2*b*Tan[c + d*x]))/d
```

Maple [A]
time = 0.24, size = 119, normalized size = 0.98

method	result
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{}{3\sin(dx+c}\right.}{d}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{}{3\sin(dx+c}\right.}{d}$
risch	$\frac{4ab\,e^{10i(dx+c)}-16ab\,e^{8i(dx+c)}-\frac{32ia^2e^{6i(dx+c)}}{3}+\frac{32ib^2e^{6i(dx+c)}}{3}-\frac{16ia^2e^{4i(dx+c)}}{3}-\frac{80ib^2e^{4i(dx+c)}}{3}+16ab\,e^{4i(dx+c)}+\frac{64ia^2e}{}}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/4/
sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+b^2*(-1/3/sin(d*x+c)^3/cos(d*
x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c)))
```

Maxima [A]
time = 0.42, size = 104, normalized size = 0.85

$$\frac{60\,ab\log\left(\tan\left(dx+c\right)\right)+30\,b^2\tan\left(dx+c\right)-\frac{60\,ab\tan(dx+c)^3+30\,\left(a^2+2\,b^2\right)\tan(dx+c)^4+15\,ab\tan(dx+c)+10\,\left(2\,a^2+b^2\right)\tan(dx+c)^2+6\,a^2}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/30*(60*a*b*log(tan(d*x + c)) + 30*b^2*tan(d*x + c) - (60*a*b*tan(d*x + c)^3 + 30*(a^2 + 2*b^2)*tan(d*x + c)^4 + 15*a*b*tan(d*x + c) + 10*(2*a^2 + b^2)*tan(d*x + c)^2 + 6*a^2)/tan(d*x + c)^5)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.
time = 0.35, size = 240, normalized size = 1.97

$$-\frac{16\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{6}-40\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{4}+30\left(a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{2}+30\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-30\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^{2}+\frac{1}{4}\right)\sin\left(dx+c\right)-30\,b^{2}-15\left(2\,ab\cos\left(dx+c\right)^{3}-3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{30\left(d\cos\left(dx+c\right)^{5}-2\,d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -1/30*(16*(a^2 + 5*b^2)*cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*cos(d*x + c)^4 + 30*(a^2 + 5*b^2)*cos(d*x + c)^2 + 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 30*b^2 - 15*(2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\csc^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**6, x)

Giac [A]
time = 0.60, size = 131, normalized size = 1.07

$$\frac{60\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+30\,b^{2}\tan\left(dx+c\right)-\frac{137\,ab\tan\left(dx+c\right)^{5}+30\,a^{2}\tan\left(dx+c\right)^{4}+60\,b^{2}\tan\left(dx+c\right)^{4}+60\,ab\tan\left(dx+c\right)^{3}+20\,a^{2}\tan\left(dx+c\right)^{2}+10\,b^{2}\tan\left(dx+c\right)^{2}+15\,ab\tan\left(dx+c\right)+6\,a^{2}}{\tan\left(dx+c\right)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d*x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x + c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c) + 6*a^2)/tan(d*x + c)^5)/d

```
c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c)
+ 6*a^2)/tan(d*x + c)^5)/d
```

Mupad [B]
time = 4.06, size = 107, normalized size = 0.88

$$\frac{b^2\tan(c+dx)}{d}-\frac{\tan(c+dx)^4\,(a^2+2\,b^2)+\frac{a^2}{5}+\tan(c+dx)^2\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\frac{a\,b\tan(c+dx)}{2}+2\,a\,b\tan(c+dx)^3}{d\tan\left(c+dx\right)^5}+\frac{2\,a\,b\,\ln\left(\tan(c+dx)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^4*(a^2 + 2*b^2) + a^2/5 + tan(c + d*x)
^2*((2*a^2)/3 + b^2/3) + (a*b*tan(c + d*x))/2 + 2*a*b*tan(c + d*x)^3)/(d*ta
n(c + d*x)^5) + (2*a*b*log(tan(c + d*x)))/d
```

3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=205

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-ab$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+6*a*b^2*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-a*b^2*cos(d*x+c)^3/d+3*a*b^2*
sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-a^2*b*sin(d*x+c)^3/d
+5/6*b^3*sin(d*x+c)^3/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276, 294}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{5b^3\sin^3(c+dx)}{6d}+\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}-\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d)
 - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d
*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin
[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^3(c+dx)+3a^2b\sin^3(c+dx)\tan(c+dx)+3ab^2\sin^3(c+\right.$$

$$= a^3\int \sin^3(c+dx)\,dx + \left(3a^2b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx + \left(3a\right.$$

$$= -\frac{a^3\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,d\right.}{d}$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(}{d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + $$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+}{d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 771 vs. 2(205) = 410.
time = 6.32, size = 771, normalized size = 3.76

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(4
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*C
os[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan
[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*T
```

```
an[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b*(3*a^2 - b^2
)*Cos[c + d*x]^3*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.20, size = 184, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3b^2a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\mathrm{s}\right.\right.}{}$
default	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3b^2a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\mathrm{s}\right.\right.}{}$
risch	$-\frac{ib^2\left(6ia\,e^{3i(dx+c)}+b\,e^{3i(dx+c)}+6ia\,e^{i(dx+c)}-e^{i(dx+c)}b\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{ie^{-3i(dx+c)}b\,a^2}{8d}+\frac{e^{3i(dx+c)}a^3}{24d}-\frac{e^{3i(dx+c)}b^2a}{8d}-\frac{9ie^{i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/3*sin(d*x+c)^3-sin(d*
x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*a*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d
*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/
2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.35, size = 173, normalized size = 0.84

$$\frac{4\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)a^3-6\left(2\sin\left(dx+c\right)^3-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)+6\sin\left(dx+c\right)\right)a^2b-12\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)ab^2+\left(4\sin\left(dx+c\right)^3-\frac{6\sin(dx+c)}{\sin(dx+c)^2-1}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+24\sin\left(dx+c\right)\right)b^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12*(
cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c)^3
 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(
sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

Fricas [A]
time = 0.35, size = 188, normalized size = 0.92

$$\frac{4\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+36\,ab^2\cos\left(dx+c\right)-12\left(a^3-6\,ab^2\right)\cos\left(dx+c\right)^3+3\left(6\,a^2b-5\,b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\left(6\,a^2b-5\,b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4+3\,b^3-2\left(12\,a^2b-7\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x +
c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan (c + dx))^3 \sin^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 72200 vs. 2(195) = 390.
time = 168.92, size = 72200, normalized size = 352.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 360*pi*a*b^2*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan
(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
```

```
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 360*pi*a*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^9*tan(1/2*c)^9 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c
)^9 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^8*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*a*b^2*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*
a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(
1/2*c)^10 + 288*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 240*b^3*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 288*a^2*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
10*tan(1/2*c)^10 + 240*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
```

```
 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 360*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -...
```

Mupad [B]
time = 6.44, size = 291, normalized size = 1.42

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(6\,a^2\,b-5\,b^3\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b-5\,b^3\right)+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-16\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(16\,a\,b^2-\frac{4\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(32\,a\,b^2-\frac{20\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(6\,a^2\,b-5\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(8\,a^2\,b-\frac{20\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(8\,a^2\,b-\frac{20\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(28\,a^2\,b-\frac{22\,b^3}{3}\right)+\frac{4\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b - 5*b^3))/d - (tan(c/2 + (d*x)/2)*(6*a^
2*b - 5*b^3) + 4*a^3*tan(c/2 + (d*x)/2)^6 - 16*a*b^2 - tan(c/2 + (d*x)/2)^2
*(16*a*b^2 - (4*a^3)/3) + tan(c/2 + (d*x)/2)^4*(32*a*b^2 - (20*a^3)/3) + ta
n(c/2 + (d*x)/2)^9*(6*a^2*b - 5*b^3) + tan(c/2 + (d*x)/2)^3*(8*a^2*b - (20*
b^3)/3) + tan(c/2 + (d*x)/2)^7*(8*a^2*b - (20*b^3)/3) - tan(c/2 + (d*x)/2)^
5*(28*a^2*b - (22*b^3)/3) + (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2
 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (
d*x)/2)^10 + 1))
```

3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=103

$$\frac{1}{2}a\left(a^2-9b^2\right)x-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{9ab^2\tan(c+dx)}{2d}+\frac{b^3\tan^2(c+dx)}{d}-\frac{\cos(c+dx)\sin(c+dx)(}{2d}$$

```
[Out] 1/2*a*(a^2-9*b^2)*x-b*(3*a^2-2*b^2)*ln(cos(d*x+c))/d+9/2*a*b^2*tan(d*x+c)/d
+b^3*tan(d*x+c)^2/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1659, 815, 649, 209, 266}

$$-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{1}{2}ax\left(a^2-9b^2\right)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d}+\frac{b^3\tan^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*
Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(
a + b*Tan[c + d*x])^3)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^3}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^2(-a}{b^2+}\right.}{}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \left(-9ab^2\right.\right.}{}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+}{2d}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+}{2d}$$

$$= \frac{1}{2}a\left(a^2-9b^2\right)x - \frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{9ab^2\tan(c+dx}{2d}$$

Mathematica [A]
time = 4.67, size = 203, normalized size = 1.97

$$\frac{b\left(-\frac{a(a^2-3b^2)\text{ArcTan}(\tan(c+dx))}{b} + (3a^2-b^2)\cos^2(c+dx) + \left(3a^2-2b^2+\frac{a^3-6ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + \left(3a^2-2b^2+\frac{-a^3+6ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right) - \frac{a(a^2-3b^2)\sin(2(c+dx))}{2b} + 6ab\tan(c+dx) + b^2\tan^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*x
]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
 + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)])/(2*b) + 6*a*b*Tan[c + d
*x] + b^2*Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.17, size = 163, normalized size = 1.58

method	result
derivativedivides	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right.\right.}{d}$
risch	$\frac{3i e^{-2i(dx+c)}b^2a}{8d}-2ix\,b^3+\frac{a^3x}{2}-\frac{9a\,b^2x}{2}+\frac{3\,e^{2i(dx+c)}b\,a^2}{8d}-\frac{e^{2i(dx+c)}b^3}{8d}-\frac{i e^{-2i(dx+c)}a^3}{8d}-\frac{4ib^3c}{d}+3\,e^{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(-1/2*sin(d*x+c
)^2-ln(cos(d*x+c)))+3*b^2*a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(
d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*si
n(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.67, size = 113, normalized size = 1.10

$$\frac{b^3\tan(dx+c)^2+6\,ab^2\tan(dx+c)+(a^3-9\,ab^2)(dx+c)+(3\,a^2b-2\,b^3)\log\left(\tan(dx+c)^2+1\right)+\frac{3\,a^2b-b^3-(a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + (a^3 - 9*a*b^2)*(d*x + c)
+ (3*a^2*b - 2*b^3)*log(tan(d*x + c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a*b
^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 149, normalized size = 1.45

$$\frac{2\,(3\,a^2b-b^3)\cos(dx+c)^4-4\,(3\,a^2b-2\,b^3)\cos(dx+c)^2\log(-\cos(dx+c))+2\,b^3-(3\,a^2b-b^3-2\,(a^3-9\,ab^2)dx)\cos(dx+c)^2+2\,(6\,ab^2\cos(dx+c)-(a^3-3\,ab^2)\cos(dx+c)^3)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 4*(3*a^2*b - 2*b^3)*cos(d*x + c)^2*
log(-cos(d*x + c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*cos(d*
x + c)^2 + 2*(6*a*b^2*cos(d*x + c) - (a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2590 vs. 2(97) = 194.
time = 1.32, size = 2590, normalized size = 25.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*d*x*tan(d*x)^4*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*a
^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
+ 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)
^4 + 2*a^3*d*x*tan(d*x)^4*tan(c)^2 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^2 - 4*a
^3*d*x*tan(d*x)^3*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a^3*d*x*t
an(d*x)^2*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 3*a^2*b*tan(d*x)^4*
tan(c)^4 + b^3*tan(d*x)^4*tan(c)^4 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 8*b^3*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*a^3*tan(d*x)^4*tan(
c)^3 - 18*a*b^2*tan(d*x)^4*tan(c)^3 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
```

c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 2*a^3*tan(d*x)^3*tan(c)^4 - 1
8*a*b^2*tan(d*x)^3*tan(c)^4 - 4*a^3*d*x*tan(d*x)^3*tan(c) + 36*a*b^2*d*x*ta
n(d*x)^3*tan(c) + 4*a^3*d*x*tan(d*x)^2*tan(c)^2 - 36*a*b^2*d*x*tan(d*x)^2*t
an(c)^2 - 3*a^2*b*tan(d*x)^4*tan(c)^2 + 5*b^3*tan(d*x)^4*tan(c)^2 - 4*a^3*d
*x*tan(d*x)*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)*tan(c)^3 - 18*a^2*b*tan(d*x)^3
*tan(c)^3 + 6*b^3*tan(d*x)^3*tan(c)^3 - 3*a^2*b*tan(d*x)^2*tan(c)^4 + 5*b^3
*tan(d*x)^2*tan(c)^4 + 12*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)^3*tan(c) - 8*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^3*tan(c) - 12*a*b^2*tan(d*x)^4*tan(c) - 12*a^2*b*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 8*b^3*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 6*a^3*tan
(d*x)^3*tan(c)^2 + 18*a*b^2*tan(d*x)^3*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 8*b^3*log(4*(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 6*a^3*tan(d*x)^2*tan
(c)^3 + 18*a*b^2*tan(d*x)^2*tan(c)^3 - 12*a*b^2*tan(d*x)*tan(c)^4 + 2*a^3*d
*x*tan(d*x)^2 - 18*a*b^2*d*x*tan(d*x)^2 + 2*b^3*tan(d*x)^4 - 4*a^3*d*x*tan(
d*x)*tan(c) + 36*a*b^2*d*x*tan(d*x)*tan(c) + 6*a^2*b*tan(d*x)^3*tan(c) - 2*
b^3*tan(d*x)^3*tan(c) + 2*a^3*d*x*tan(c)^2 - 18*a*b^2*d*x*tan(c)^2 + 30*a^2
*b*tan(d*x)^2*tan(c)^2 - 2*b^3*tan(d*x)^2*tan(c)^2 + 6*a^2*b*tan(d*x)*tan(c
)^3 - 2*b^3*tan(d*x)*tan(c)^3 + 2*b^3*tan(c)^4 - 6*a^2*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 12*a*b^2*tan(d*x)^3 + 12*a^2*b*log(4
*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 8*b^3*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 6*a^3*tan(d*x
)^2*tan(c) - 18*a*b^2*tan(d*x)^2*tan(c) - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^
2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan
(c) + 1)/(tan(c)^2 + 1))*tan(c)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(c)^2 + 6*a^3*tan(d*x)*tan(c)^2 - 18*a*b^2*tan(d*x)*tan
(c)^2 + 12*a*b^2*tan(c)^3 + 2*a^3*d*x - 18*a*b^2*d*x - 3*a^2*b*tan(d*x)^2 +
 5*b^3*tan(d*x)^2 - 18*a^2*b*tan(d*x)*tan(c) + 6*b^3*tan(d*x)*tan(c) - 3*a^
2*b*tan(c)^2 + 5*b^3*tan(c)^2 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(c)^2 + 1)) + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) -

```
2*a^3*tan(d*x) + 18*a*b^2*tan(d*x) - 2*a^3*tan(...
```

Mupad [B]
time = 3.70, size = 151, normalized size = 1.47

$$\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{\cos\left(c+d\,x\right)^2\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}+\tan(c+d\,x)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-b^3\right)}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(c+d\,x)\,(a-3\,b)\,(a+3\,b)}{2\,\left(\frac{9\,a\,b^2}{2}-\frac{a^3}{2}\right)}\right)\,(a-3\,b)\,(a+3\,b)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) + (cos(c + d*x)^2*((3*a^2*b)/2 - b^3/2 + tan(c +
 d*x)*((3*a*b^2)/2 - a^3/2)))/d + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 - b
^3))/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a - 3*b)*(a +
3*b))/(2*((9*a*b^2)/2 - a^3/2)))*(a - 3*b)*(a + 3*b))/(2*d)
```

3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-3a$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+3*a*b^2*cos(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/2*b^3*si
n(d*x+c)/d+1/2*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3598, 2718, 2672, 327, 212, 2670, 14, 294}

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3b^3\sin(c+dx)}{2d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}-\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d -
 (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*
Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin(c+dx)+3a^2b\sin(c+dx)\tan(c+dx)+3ab^2\sin(c+dx)\,\mathrm{t}\right.$$

$$= a^3\int \sin(c+dx)\,dx + \left(3a^2b\right)\int \sin(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right)$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(3ab^2\right)}{}$$

$$= -\frac{a^3\cos(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d} +$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\cos(c+dx)}{d} + \frac{3ab}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 637 vs. 2(133) = 266.
time = 6.17, size = 637, normalized size = 4.79

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
+ (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b*(3*a^2 - b^2)*Cos[c + d*x]^
3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)
```

Maple [A]
time = 0.21, size = 144, normalized size = 1.08

method	result
derivativedivides	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3b^2a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
default	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3b^2a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
risch	$\frac{3i\,e^{i(dx+c)}b\,a^2}{2d}-\frac{i\,e^{i(dx+c)}b^3}{2d}-\frac{e^{i(dx+c)}a^3}{2d}+\frac{3\,e^{i(dx+c)}b^2a}{2d}-\frac{3i\,e^{-i(dx+c)}b\,a^2}{2d}+\frac{i\,e^{-i(dx+c)}b^3}{2d}-\frac{e^{-i(dx+c)}a^3}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*cos(d*x+c)+3*a^2*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*
a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)
^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

Maxima [A]
time = 0.43, size = 128, normalized size = 0.96

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-12\,ab^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-6\,a^2b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))+4\,a^3\cos(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 12*a*b^2*(1/cos(d*x + c) + cos(
d*x + c)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 4*a^3*cos(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 144, normalized size = 1.08

$$\frac{12\,ab^2\cos(dx+c)-4\,(a^3-3\,ab^2)\cos(dx+c)^3+3\,(2\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-3\,(2\,a^2b-b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(b^3-2\,(3\,a^2b-b^3)\cos(dx+c)^2)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(2*a^2*b
- b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^2*b - b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14636 vs. $2(127) = 254$.
time = 9.24, size = 14636, normalized size = 110.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*a^2*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
6*tan(1/2*c)^6 + 3*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b^3*log(2*
(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)
^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^6 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 72
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 - 9*pi*a*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 4*a^3*tan(1/2*d*x)^6*tan(
```

```
1/2*c)^6 + 24*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a^2*b*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 3
*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^6*tan(1/2*c)^4 - 6*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*b^3*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 48*a^2*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
5*tan(1/2*c)^5 - 24*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 48*a^2*b*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 + 24*b^3*log
(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d
*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*ta
n(1/2*c)^5 + 24*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^5 - 12*b^3*tan(1/2*d*x)^6*t
an(1/2*c)^5 + 6*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 3*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d...
```

Mupad [B]

time = 5.80, size = 193, normalized size = 1.45

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2b-3b^3\right)+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12ab^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12ab^2-4a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(6a^2b-3b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(12a^2b-2b^3\right)+2a^3}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(6a^2b-3b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*a^2*b - 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 - 12*a*b
^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*
b - 3*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 2*b^3) + 2*a^3)/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1)) + (atanh(ta
n(c/2 + (d*x)/2))*(6*a^2*b - 3*b^3))/d
```

3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec(c+$$

```
[Out] -a^3*arctanh(cos(d*x+c))/d+3*a^2*b*arctanh(sin(d*x+c))/d-1/2*b^3*arctanh(si
n(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3598, 3855, 2686, 8, 2691}

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3
*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx &= \int \left(a^3\csc(c+dx)+3a^2b\sec(c+dx)+3ab^2\sec(c+dx)\tan(c+dx)\right. \\
&= a^3\int \csc(c+dx)\,dx+\left(3a^2b\right)\int \sec(c+dx)\,dx+\left(3ab^2\right)\int \sec(c+d \\
&= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\sec(c+dx}{} \\
&= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^3\tanh^{-1}(\mathrm{si}}{2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(86) = 172.
time = 2.37, size = 241, normalized size = 2.80

$$\frac{12ab^2-4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+24ab^2\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*L
og[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)
```

Maple [A]
time = 0.26, size = 107, normalized size = 1.24

method	result
derivativedivides	$\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+\frac{3b^2a}{\cos(dx+c)}+b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\mathrm{tan}}{2}\right.}{d}$

default	$\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+\frac{3b^2a}{\cos(dx+c)}+b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+t\cdots}{2}\cdots\right.}{d}$
risch	$-\frac{ib^2\left(6ia\,e^{3i(dx+c)}+b\,e^{3i(dx+c)}+6ia\,e^{i(dx+c)}-e^{i(dx+c)}b\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{3a^2b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{3a^2b\ln(e\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*b^2*
a/cos(d*x+c)+b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d
*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.37, size = 111, normalized size = 1.29

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-6a^2b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4a^3\log(\cot(dx+c)+\csc(dx+c))-\frac{12ab^2}{\cos(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 4*a^3*log(cot(d*x + c) + csc(d*x + c)) - 12*a*b^2/cos(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 148, normalized size = 1.72

$$-\frac{2a^3\cos(dx+c)^2\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2a^3\cos(dx+c)^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-12ab^2\cos(dx+c)-(6a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)+(6a^2b-b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)-2b^3\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c)
^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))^3\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)
```

Giac [A]
time = 0.79, size = 144, normalized size = 1.67

$$\frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/
2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 4.21, size = 278, normalized size = 3.23

$$\frac{2\left(\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{b^3\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,1\mathrm{i}}{2}+a^2\,b\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,3\mathrm{i}\right)}{d}+\frac{\frac{\sin\left(c+d\,x\right)\,b^3}{2}+3\,a\,\cos\left(c+d\,x\right)\,b^2}{d\,\left(\frac{\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (2*((a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^3*atan((b^3*cos
(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a
^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x
)/2)*6i))*1i)/2 + a^2*b*atan((b^3*cos(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x
)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2
 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x)/2)*6i))*3i))/d + ((b^3*sin(c + d*x))
/2 + 3*a*b^2*cos(c + d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[Out] -a^3*cot(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan
(d*x+c)^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 45}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*Cot[c + d*x])/d) + (3*a^2*b*Log[Tan[c + d*x]])/d + (3*a*b^2*Tan[c +
d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)\,dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3}{}\end{aligned}$$

Mathematica [A]
time = 1.05, size = 126, normalized size = 1.97

$$-\frac{\csc(c+dx)\sec^2(c+dx)\left(3a(a^2-b^2)\cos(c+dx)+(a^3+3ab^2)\cos(3(c+dx))-2b(b^2-3a^2\log(\cos(c+dx))-3a^2\cos(2(c+dx))(\log(\cos(c+dx))-\log(\sin(c+dx)))+3a^2\log(\sin(c+dx)))\sin(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/4*(Csc[c + d*x]*Sec[c + d*x]^2*(3*a*(a^2 - b^2)*Cos[c + d*x] + (a^3 + 3*
a*b^2)*Cos[3*(c + d*x)] - 2*b*(b^2 - 3*a^2*Log[Cos[c + d*x]] - 3*a^2*Cos[2*
(c + d*x)]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) + 3*a^2*Log[Sin[c + d*x]
])*Sin[c + d*x]))/d
```

Maple [A]
time = 0.23, size = 55, normalized size = 0.86

method	result
derivativedivides	$\frac{-a^3\cot(dx+c)+3a^2b\ln(\tan(dx+c))+3b^2a\tan(dx+c)+\frac{b^3}{2\cos(dx+c)^2}}{d}$
default	$\frac{-a^3\cot(dx+c)+3a^2b\ln(\tan(dx+c))+3b^2a\tan(dx+c)+\frac{b^3}{2\cos(dx+c)^2}}{d}$
risch	$\frac{-2ia^3e^{4i(dx+c)}+6ia\,b^2e^{4i(dx+c)}+2b^3e^{4i(dx+c)}-4ia^3e^{2i(dx+c)}-2b^3e^{2i(dx+c)}-2ia^3-6ia\,b^2}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}-1\right)}+\frac{3a^2b\ln\left(e^{2i(dx+c)}-1\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*cot(d*x+c)+3*a^2*b*ln(tan(d*x+c))+3*b^2*a*tan(d*x+c)+1/2*b^3/cos(
d*x+c)^2)
```

Maxima [A]
time = 0.43, size = 56, normalized size = 0.88

$$\frac{b^3\tan(dx+c)^2+6\,a^2b\log(\tan(dx+c))+6\,ab^2\tan(dx+c)-\frac{2\,a^3}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(tan(d*x + c)) + 6*a*b^2*tan(d*x + c)
- 2*a^3/tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(62) = 124$.
time = 0.34, size = 127, normalized size = 1.98

$$-\frac{3\,a^2b\cos(dx+c)^2\log\left(\cos(dx+c)^2\right)\sin(dx+c)-3\,a^2b\cos(dx+c)^2\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)-6\,ab^2\cos(dx+c)+2\left(a^3+3\,ab^2\right)\cos(dx+c)^3-b^3\sin(dx+c)}{2\,d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)^2*log(cos(d*x + c)^2)*sin(d*x + c) - 3*a^2*b*cos
(d*x + c)^2*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^2*cos(d*x +
 c) + 2*(a^3 + 3*a*b^2)*cos(d*x + c)^3 - b^3*sin(d*x + c))/(d*cos(d*x + c)^
2*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.77, size = 70, normalized size = 1.09

$$\frac{b^3\tan(dx+c)^2+6\,a^2b\log(|\tan(dx+c)|)+6\,ab^2\tan(dx+c)-\frac{2\,(3\,a^2b\tan(dx+c)+a^3)}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x
+ c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d
```

Mupad [B]
time = 3.66, size = 62, normalized size = 0.97

$$\frac{b^3\tan(c+d\,x)^2}{2\,d}-\frac{a^3\cot(c+d\,x)}{d}+\frac{3\,a^2\,b\,\ln(\tan(c+d\,x))}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) - (a^3*cot(c + d*x))/d + (3*a^2*b*log(tan(c + d*
x)))/d + (3*a*b^2*tan(c + d*x))/d
```

3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=141

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a}{}$$

```
[Out] -1/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arctan
h(sin(d*x+c))/d+1/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-1/2*a^3*
cot(d*x+c)*csc(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/
d
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3598, 3853, 3855, 2701, 327, 213, 2702}

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*ArcTanh[Cos[c + d*x]])/d - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (3
*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^
2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*Sec[
c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx &= \int \left(a^3\csc^3(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc(c+\right.\\ &= a^3\int \csc^3(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(3a\right.\\ &= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a^3\int\\ &= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b\csc(c+}{d}\\ &= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh}{}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 897 vs. 2(141) = 282.

time = 6.19, size = 897, normalized size = 6.36

$$\frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{3a^2b\cos^3(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a^3\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{(-a^3-6ab^2)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{(-6a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{(a^3+6ab^2)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{(6a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a^3\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{3a^2b\cos^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x]
)^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
 + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c +
 d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^3)*Cos
[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3
)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c +
 d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c +
d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2
*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a^2*b*Cos[
c + d*x]^3*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.27, size = 140, normalized size = 0.99

method	result
derivativedivides	$\frac{a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3b^2a\left(\frac{1}{\cos(dx+c)}+\ln\right.}{d}$
default	$\frac{a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3b^2a\left(\frac{1}{\cos(dx+c)}+\ln\right.}{d}$
risch	$-\frac{i\left(3ia^3e^{3i(dx+c)}+6ia\,b^2e^{i(dx+c)}+6a^2b\,e^{7i(dx+c)}+b^3e^{7i(dx+c)}+6ia\,b^2e^{7i(dx+c)}-6ia\,b^2e^{5i(dx+c)}+6a^2b\,e^{5i(dx+c)}-3b^3e^{5i}\right.}{d\left(e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b
*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*a*(1/cos(d*x+c)+ln(csc(d*x
```

```
+c)-cot(d*x+c)))+b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

Maxima [A]
time = 0.63, size = 171, normalized size = 1.21

$$\frac{a^{3}\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-b^{3}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,ab^{2}\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,a^{2}b\left(\frac{2}{\sin\left(dx+c\right)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos(d*
x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(133) = 266.
time = 0.40, size = 299, normalized size = 2.12

$$-\frac{12\,ab^{2}\cos\left(dx+c\right)-2\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{3}+\left(\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{4}-\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{4}-\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\left(6\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}-\left(6\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(\left(6\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}-\left(6\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(b^{3}-\left(6\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^{4}-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6*
a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2
)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a^2*b
 + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d*x +
c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(b^3 - (6
*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x +
 c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\csc^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(133) = 266.

time = 0.81, size = 304, normalized size = 2.16

$$\frac{a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 4 \, {\left(6 \, a^{2} b + b^{3}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 1 \right|}\right) - 4 \, {\left(6 \, a^{2} b + b^{3}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 1 \right|}\right) + 4 \, {\left(a^{3} + 6 \, a b^{2}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) - \frac{2 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 12 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} - 8 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} - 3 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 24 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 24 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 8 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 36 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + a^{3}}{{\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right)}^{2}}}{8 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 4*(6*a^2*
b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*(6*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))
- (2*a^3*tan(1/2*d*x + 1/2*c)^6 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*
b*tan(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^3*tan(1/2*d*x
 + 1/2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 8*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*
a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1
/2*c))^2)/d
```

Mupad [B]

time = 3.97, size = 581, normalized size = 4.12

$$\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^3}{2}+24\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^3+24\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\,b-4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\,a^2\,b+4\,b^3\right)+\frac{a^3}{2}+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^3}{2}+3\,a\,b^2\right)}{d}-\frac{\mathrm{atan}\left(-\frac{\left(3\,a^2\,b+\frac{b^3}{2}\right)\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{b^3}{2}\right)+6\,a^2\,b+b^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+6\,a\,b^2\right)\right)\,1{}\mathrm{i}-\left(3\,a^2\,b+\frac{b^3}{2}\right)\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{b^3}{2}\right)-6\,a^2\,b-b^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+6\,a\,b^2\right)\right)\,1{}\mathrm{i}}{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(36\,a^4\,b^2+12\,a^2\,b^4+b^6\right)-\left(3\,a^2\,b+\frac{b^3}{2}\right)\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{b^3}{2}\right)+6\,a^2\,b+b^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+6\,a\,b^2\right)\right)-\left(3\,a^2\,b+\frac{b^3}{2}\right)\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{b^3}{2}\right)-6\,a^2\,b-b^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+6\,a\,b^2\right)\right)+6\,a\,b^5+6\,a^5\,b+37\,a^3\,b^3}\right)\,\left(a^2\,b\,6{}\mathrm{i}+b^3\,1{}\mathrm{i}\right)}{d}-\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (tan(c/2 + (d*x)/2)^4*(24*a*b^2 + a^3/2)
 - tan(c/2 + (d*x)/2)^2*(24*a*b^2 + a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b -
4*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + 4*b^3) + a^3/2 + 6*a^2*b*tan(c/2
+ (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
 + (d*x)/2)^6)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + a^3/2))/d - (atan(-((
3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 -
tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3))*1i - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*
x)/2)*(3*a^2*b + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3
))*1i)/(2*tan(c/2 + (d*x)/2)*(b^6 + 12*a^2*b^4 + 36*a^4*b^2) - (3*a^2*b + b
^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 - tan(c/2 + (
d*x)/2)*(6*a*b^2 + a^3)) - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b
 + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3)) + 6*a*b^5 +
 6*a^5*b + 37*a^3*b^3))*(a^2*b*6i + b^3*1i))/d - (3*a^2*b*tan(c/2 + (d*x)/2
))/(2*d)
```

3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=113

$$-\frac{a(a^2+3b^2)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b(3a^2+b^2)\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+$$

[Out] -a*(a^2+3*b^2)*cot(d*x+c)/d-3/2*a^2*b*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d +b*(3*a^2+b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d

Rubi [A]

time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a(a^2+3b^2)\cot(c+dx)}{d}+\frac{b(3a^2+b^2)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]

[Out] -((a*(a^2 + 3*b^2)*Cot[c + d*x])/d) - (3*a^2*b*Cot[c + d*x]^2)/(2*d) - (a^3 *Cot[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Log[Tan[c + d*x]])/d + (3*a*b^2*T an[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)}{x^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(3a+\frac{a^3b^2}{x^4}+\frac{3a^2b^2}{x^3}+\frac{a^3+3ab^2}{x^2}+\frac{3a^2+b^2}{x}+x\right)dx,x,b\tan(c\right.}{d}\\ &= -\frac{a(a^2+3b^2)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\end{aligned}$$

Mathematica [A]
time = 2.30, size = 212, normalized size = 1.88

$$\frac{(b+a\cot(c+dx))^3\sec^2(c+dx)\left(-16a^3\cos(c+dx)-2\sin(c+dx)\left(18a^2b-6b^3+6(3a^2b+b^3)\cos(2(c+dx))+9a^2b\log(\cos(c+dx))+3b^3\log(\cos(c+dx))-3b(3a^2+b^2)\cos(4(c+dx))(\log(\cos(c+dx))-\log(\sin(c+dx)))-9a^2b\log(\sin(c+dx))-3b^3\log(\sin(c+dx))+2a^3\sin(4(c+dx))+18ab^2\sin(4(c+dx))\right)\right)}{48d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Sec[c + d*x]^2*(-16*a^3*Cos[c + d*x] - 2*Sin[c + d*
x]*(18*a^2*b - 6*b^3 + 6*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + 9*a^2*b*Log[Cos
[c + d*x]] + 3*b^3*Log[Cos[c + d*x]] - 3*b*(3*a^2 + b^2)*Cos[4*(c + d*x)]*(
Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) - 9*a^2*b*Log[Sin[c + d*x]] - 3*b^3*
Log[Sin[c + d*x]] + 2*a^3*Sin[4*(c + d*x)] + 18*a*b^2*Sin[4*(c + d*x)])))/(
48*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.26, size = 106, normalized size = 0.94

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3b^2a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3 \ldots}{d}$
default	$\frac{a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3b^2a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3 \ldots}{d}$
risch	$\frac{6a^2b\,e^{8i(dx+c)}+2b^3e^{8i(dx+c)}-12ia\,b^2e^{6i(dx+c)}-12ia\,b^2+6a^2b\,e^{6i(dx+c)}-6b^3e^{6i(dx+c)}+\frac{4ia^3e^{2i(dx+c)}}{3}+4ia^3e^{6i(dx+c)}-6a \ldots}{d\left(e^{2i(dx+c)}+1\right)^2\left(e \ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1/2/sin(d*x+c)^2+ln(t
an(d*x+c)))+3*b^2*a*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b^3*(1/2/cos(d*x
+c)^2+ln(tan(d*x+c))))
```

Maxima [A]
time = 0.35, size = 98, normalized size = 0.87

$$\frac{3\,b^3\tan\left(dx+c\right)^2+18\,ab^2\tan\left(dx+c\right)+6\left(3\,a^2b+b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{9\,a^2b\tan(dx+c)+2\,a^3+6\,\left(a^3+3\,ab^2\right)\tan(dx+c)^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(t
an(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 + 6*(a^3 + 3*a*b^2)*tan(d*x +
c)^2)/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(107) = 214$.
time = 0.35, size = 237, normalized size = 2.10

$$-\frac{4\,(a^3+9\,ab^2)\cos(dx+c)^5+18\,ab^2\cos(dx+c)-6\,(a^3+9\,ab^2)\cos(dx+c)^3+3\,\left((3\,a^2b+b^3)\cos(dx+c)^4-(3\,a^2b+b^3)\cos(dx+c)^2\right)\log\left(\cos(dx+c)^2\right)\sin(dx+c)-3\,\left((3\,a^2b+b^3)\cos(dx+c)^4-(3\,a^2b+b^3)\cos(dx+c)^2\right)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)+3\,\left(b^3-(3\,a^2b+b^3)\cos(dx+c)^2\right)\sin(dx+c)}{6\,\left(d\cos(dx+c)^4-d\cos(dx+c)^2\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(a^3 + 9*a*b^2)*cos(d*x + c)^5 + 18*a*b^2*cos(d*x + c) - 6*(a^3 + 9
*a*b^2)*cos(d*x + c)^3 + 3*((3*a^2*b + b^3)*cos(d*x + c)^4 - (3*a^2*b + b^3
)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*((3*a^2*b + b^3)*cos
(d*x + c)^4 - (3*a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4
)*sin(d*x + c) + 3*(b^3 - (3*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/((d
*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\csc^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**4, x)
```

Giac [A]
time = 0.81, size = 133, normalized size = 1.18

$$\frac{3\,b^3\tan(dx+c)^2+18\,ab^2\tan(dx+c)+6\,(3\,a^2b+b^3)\log(|\tan(dx+c)|)-\frac{33\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3+6\,a^3\tan(dx+c)^2+18\,ab^2\tan(dx+c)^2+9\,a^2b\tan(dx+c)+2\,a^3}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(a
bs(tan(d*x + c))) - (33*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x + c)^3 + 6*a^
3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c)^2 + 9*a^2*b*tan(d*x + c) + 2*a^3)/
tan(d*x + c)^3)/d
```

Mupad [B]
time = 3.72, size = 103, normalized size = 0.91

$$\frac{\ln(\tan(c+d\,x))\,(3\,a^2\,b+b^3)}{d}-\frac{\cot(c+d\,x)^3\,\left(\frac{a^3}{3}+\tan(c+d\,x)^2\,(a^3+3\,a\,b^2)+\frac{3\,a^2\,b\tan(c+d\,x)}{2}\right)}{d}+\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + b^3))/d - (cot(c + d*x)^3*(a^3/3 + tan(c + d*
x)^2*(3*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/2))/d + (b^3*tan(c + d*x)^2)/
(2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9ab^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -3/8*a^3*arctanh(cos(d*x+c))/d-9/2*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arct
anh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-3/2*b^
3*csc(d*x+c)/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-1/4*a^3
*cot(d*x+c)*csc(d*x+c)^3/d+9/2*a*b^2*sec(d*x+c)/d-3/2*a*b^2*csc(d*x+c)^2*se
c(d*x+c)/d+1/2*b^3*csc(d*x+c)*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{9ab^2\sec(c+dx)}{2d}-\frac{9ab^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}-\frac{3b^3\csc(c+dx)}{2d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*d
) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c + d
*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*Csc[
c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc^5(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^3(c+\right.$$

$$= a^3\int \csc^5(c+dx)\,dx + \left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx + (3a$$

$$= -\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}\left(3a^3\right)\int \csc^3(c+dx)\,dx - \frac{(3a^2b}{}$$

$$= -\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3ab}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} -$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{9ab^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{3a^2b\tan}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1229 vs. $2(229) = 458$.
time = 6.20, size = 1229, normalized size = 5.37

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) + ((-7*a^2*b*Cos[(c + d*x)/2] - 2*b^3*Cos[(c + d*x)/2])*Cos[c
+ d*x]^3*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a
+ b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Co
s[c + d*x]^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c + d*x)
/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) -
(3*(a^3 + 12*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b + b^3)*Cos[c
+ d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(
2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (3*(2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^3) + (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[
c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a^3*Cos[c + d*x]
^3*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin
```

```
[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b
^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c
+ d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c +
d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (Cos[c + d*x]^3*Sec[(c + d*x)/2]*(-7*a
^2*b*Sin[(c + d*x)/2] - 2*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^3)/(4*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Cos[c + d*x]^3*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)
```

Maple [A]
time = 0.30, size = 198, normalized size = 0.86

method	result
derivativedivides	$\frac{a^{3}\left(\left(-\frac{\left(\csc^{3}(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^{2}b\left(-\frac{1}{3\sin(dx+c)^{3}}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+\right.}{}$
default	$\frac{a^{3}\left(\left(-\frac{\left(\csc^{3}(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^{2}b\left(-\frac{1}{3\sin(dx+c)^{3}}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+\right.}{}$
risch	$-\frac{i\left(-30ia^{3}e^{7i(dx+c)}-5ia^{3}e^{3i(dx+c)}+24a^{2}b\,e^{11i(dx+c)}+12b^{3}e^{11i(dx+c)}+24ia\,b^{2}e^{7i(dx+c)}+3ia^{3}e^{i(dx+c)}-56a^{2}b\,e^{9i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-c
ot(d*x+c)))+3*a^2*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c
)))+3*b^2*a*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-
cot(d*x+c)))+b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x
+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.35, size = 250, normalized size = 1.09

$$\frac{a^{3}\left(\frac{2\left(3\cos(dx+c)^{3}-5\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+12\,ab^{2}\left(\frac{2\left(3\cos(dx+c)^{2}-2\right)}{\cos(dx+c)^{3}-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-4\,b^{3}\left(\frac{2\left(3\sin(dx+c)^{2}-2\right)}{\sin(dx+c)^{3}-\sin(dx+c)}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-8\,a^{2}b\left(\frac{2\left(3\sin(dx+c)^{2}+1\right)}{\sin(dx+c)^{3}}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*a*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
```

```
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b^3*(2*(3*sin(d*x + c)^2 - 2)/(s
in(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 8*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(211) = 422$.
time = 0.44, size = 427, normalized size = 1.86

$$\frac{6\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{5}+48\,ab^{2}\cos\left(dx+c\right)-10\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{3}-3\left(\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{6}-2\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{4}+\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{6}-2\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{4}+\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+12\left(\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{6}-2\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}+\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-12\left(\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{6}-2\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}+\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+8\left(3\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}+b^{3}-4\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^{6}-2\,d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 48*a*b^2*cos(d*x + c) - 10*(a^3 +
 12*a*b^2)*cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 1
2*a*b^2)*cos(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x
+ c) + 1/2) + 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*cos(d
*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) +
 12*((2*a^2*b + b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2
*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 12*((2*a^2*b + b^3)*c
os(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2*a^2*b + b^3)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*cos(d*x + c)^4 + b^3
- 4*(2*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - 2*d*c
os(d*x + c)^4 + d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\csc^{5}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**5, x)
```

Giac [**A**]
time = 0.84, size = 373, normalized size = 1.63

$$\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-120\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-32\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\left(2\,a^{2}b+b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-96\left(2\,a^{2}b+b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+24\left(a^{3}+12\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{64\left(b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}-\frac{50\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+600\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+120\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+32\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
```

```
*d*x + 1/2*c) - 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b + b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 96*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/
2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x + 1/2*c
) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^
4 + 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3
2*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(
1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

Mupad [B]
time = 4.04, size = 698, normalized size = 3.05

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^3}{8}+\frac{3\,a\,b^2}{8}\right)}{d}-\frac{\mathrm{atan}\left(\frac{\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-6\,a^2\,b-3\,b^3\right)1{}\mathrm{i}-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\,a^2\,b+3\,b^3\right)1{}\mathrm{i}}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(36\,a^4\,b^2+36\,a^2\,b^4+9\,b^6\right)+27\,a\,b^5+\frac{9\,a^5\,b}{2}-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-6\,a^2\,b-3\,b^3\right)-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\,a^2\,b+3\,b^3\right)+\frac{225\,a^3\,b^3}{4}}\right)\left(a^2\,b\,6{}\mathrm{i}+b^3\,3{}\mathrm{i}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a^3}{2}+6\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(2\,a^3+102\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{15\,a^3}{4}+108\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(26\,a^2\,b+8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(30\,a^2\,b-8\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(58\,a^2\,b+32\,b^3\right)+\frac{a^3}{4}+2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{15\,a^2\,b}{8}+\frac{b^3}{2}\right)}{d}+\frac{3\,a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^2+12\,b^2\right)}{8\,d}-\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a^
3/8))/d - (atan(((3*a^2*b + (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^
3)/4) + 6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3)*1i -
(3*a^2*b + (3*b^3)/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2
 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*a^2*b + 3*b^3)*1i)/(2*tan(c/2 + (d*x)
/2)*(9*b^6 + 36*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (3*a^2*b +
 (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*tan(c/2 + (d*x)/2
)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3) - (3*a^2*b + (3*b^3)/2)*(6*tan(c
/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)
/4) + 6*a^2*b + 3*b^3) + (225*a^3*b^3)/4))*(a^2*b*6i + b^3*3i))/d - (tan(c/
2 + (d*x)/2)^2*(6*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(102*a*b^2 + 2*
a^3) - tan(c/2 + (d*x)/2)^4*(108*a*b^2 + (15*a^3)/4) + tan(c/2 + (d*x)/2)^3
*(26*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) - tan(c/2 + (
d*x)/2)^5*(58*a^2*b + 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*
tan(c/2 + (d*x)/2)^4 - 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8))
- (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 + b^3/2))/d + (3*a*log(tan(c/2 + (d*x)/
2))*(a^2 + 12*b^2))/(8*d) - (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=167

$$-\frac{a(a^2+6b^2)\cot(c+dx)}{d}-\frac{b(6a^2+b^2)\cot^2(c+dx)}{2d}-\frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{3a^2b\cot^4(c+dx)}{4d}-\frac{a^3\cot^5}{}$$

```
[Out] -a*(a^2+6*b^2)*cot(d*x+c)/d-1/2*b*(6*a^2+b^2)*cot(d*x+c)^2/d-1/3*a*(2*a^2+3
*b^2)*cot(d*x+c)^3/d-3/4*a^2*b*cot(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d+b*(3*a
^2+2*b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 962}

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{b(6a^2+b^2)\cot^2(c+dx)}{2d}-\frac{a(a^2+6b^2)\cot(c+dx)}{d}+\frac{b(3a^2+2b^2)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^4(c+dx)}{4d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
 (3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(3a+\frac{a^3b^4}{x^6}+\frac{3a^2b^4}{x^5}+\frac{2a^3b^2+3ab^4}{x^4}+\frac{6a^2b^2+b^4}{x^3}+\frac{a^3+6ab^2}{x^2}+\frac{3a^2+}{x}\right.\right.}{d}$$

$$= -\frac{a(a^2+6b^2)\cot(c+dx)}{d}-\frac{b(6a^2+b^2)\cot^2(c+dx)}{2d}-\frac{a(2a^2+3b^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(167) = 334$.

time = 1.90, size = 515, normalized size = 3.08

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*Sec[c + d*x]^2*(40*a*(5*a^2 + 3*b^2)*Cos[c + d*x] +
8*(a^3 + 15*a*b^2)*Cos[3*(c + d*x)] - 24*a^3*Cos[5*(c + d*x)] - 360*a*b^2*C
os[5*(c + d*x)] + 8*a^3*Cos[7*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 360
*a^2*b*Sin[c + d*x] - 240*b^3*Sin[c + d*x] + 225*a^2*b*Log[Cos[c + d*x]]*Si
n[c + d*x] + 150*b^3*Log[Cos[c + d*x]]*Sin[c + d*x] - 225*a^2*b*Log[Sin[c +
 d*x]]*Sin[c + d*x] - 150*b^3*Log[Sin[c + d*x]]*Sin[c + d*x] + 270*a^2*b*Si
n[3*(c + d*x)] + 180*b^3*Sin[3*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[
3*(c + d*x)] + 30*b^3*Log[Cos[c + d*x]]*Sin[3*(c + d*x)] - 45*a^2*b*Log[Sin
[c + d*x]]*Sin[3*(c + d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[3*(c + d*x)] - 9
0*a^2*b*Sin[5*(c + d*x)] - 60*b^3*Sin[5*(c + d*x)] - 135*a^2*b*Log[Cos[c +
d*x]]*Sin[5*(c + d*x)] - 90*b^3*Log[Cos[c + d*x]]*Sin[5*(c + d*x)] + 135*a^
2*b*Log[Sin[c + d*x]]*Sin[5*(c + d*x)] + 90*b^3*Log[Sin[c + d*x]]*Sin[5*(c
+ d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] + 30*b^3*Log[Cos[c +
d*x]]*Sin[7*(c + d*x)] - 45*a^2*b*Log[Sin[c + d*x]]*Sin[7*(c + d*x)] - 30*b
^3*Log[Sin[c + d*x]]*Sin[7*(c + d*x)]))/d
```

Maple [A]

time = 0.27, size = 165, normalized size = 0.99

method	result
derivativedivides	$\frac{a^3\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3b^2a\left(-\frac{}{3\sin(d}\right.}{d}$

default	$\frac{a^{3}\left(-\frac{8}{15}-\frac{\left(\csc^{4}(dx+c)\right)}{5}-\frac{4\left(\csc^{2}(dx+c)\right)}{15}\right)\cot(dx+c)+3a^{2}b\left(-\frac{1}{4\sin(dx+c)^{4}}-\frac{1}{2\sin(dx+c)^{2}}+\ln(\tan(dx+c))\right)+3b^{2}a\left(-\frac{}{3\sin}\right.}{d}$
risch	$6a^{2}b\,e^{12i(dx+c)}+4b^{3}e^{12i(dx+c)}-18a^{2}b\,e^{10i(dx+c)}-12b^{3}e^{10i(dx+c)}-16ia\,b^{2}e^{4i(dx+c)}+\frac{16ia^{3}e^{2i(dx+c)}}{5}-24a^{2}b\,e^{8i(dx+c)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1/
4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+3*b^2*a*(-1/3/sin(d*x+c)^3/
cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+b^3*(1/2/sin(d*x+c)^2/
cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c))))
```

Maxima [A]
time = 0.46, size = 142, normalized size = 0.85

$$\frac{30\,b^{3}\tan\left(dx+c\right)^{2}+180\,ab^{2}\tan\left(dx+c\right)+60\left(3\,a^{2}b+2\,b^{3}\right)\log\left(\tan\left(dx+c\right)\right)-\frac{60\left(a^{3}+6\,ab^{2}\right)\tan(dx+c)^{4}+45\,a^{2}b\tan(dx+c)+30\left(6\,a^{2}b+b^{3}\right)\tan(dx+c)^{3}+12\,a^{3}+20\left(2\,a^{3}+3\,ab^{2}\right)\tan(dx+c)^{2}}{\tan(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*tan(d*x + c)^4 + 45*a^2*b*tan(d*x
+ c) + 30*(6*a^2*b + b^3)*tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)*ta
n(d*x + c)^2)/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(157) = 314$.
time = 0.37, size = 343, normalized size = 2.05

$$-\frac{32\left(a^{3}+15\,ab^{2}\right)\cos\left(dx+c\right)^{7}-80\left(a^{3}+15\,ab^{2}\right)\cos\left(dx+c\right)^{5}-180\,ab^{2}\cos\left(dx+c\right)+60\left(a^{3}+15\,ab^{2}\right)\cos\left(dx+c\right)^{3}+30\left(\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{6}-2\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{4}+\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)-30\left(\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{6}-2\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{4}+\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^{2}+\frac{1}{4}\right)\sin\left(dx+c\right)-15\left(2\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{4}+2\,b^{3}-3\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^{6}-2\,d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 15*a*b^2)*cos(d*x + c)^7 - 80*(a^3 + 15*a*b^2)*cos(d*x + c
)^5 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 15*a*b^2)*cos(d*x + c)^3 + 30*((3*
a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2
*b + 2*b^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((3*a^2*b
 + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2*b +
2*b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*cos(d*x + c)
^2)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x + c)^
2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(c + dx))^3 \csc^6(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**6, x)
```

Giac [A]
time = 0.86, size = 189, normalized size = 1.13

$$\frac{30\,b^3\tan(dx+c)^2 + 180\,ab^2\tan(dx+c) + 60\,(3\,a^2b + 2\,b^3)\log(|\tan(dx+c)|) - \frac{411\,a^2b\tan(dx+c)^5 + 274\,b^3\tan(dx+c)^5 + 60\,a^3\tan(dx+c)^4 + 360\,ab^2\tan(dx+c)^4 + 180\,a^2b\tan(dx+c)^3 + 30\,b^3\tan(dx+c)^3 + 40\,a^3\tan(dx+c)^2 + 60\,ab^2\tan(dx+c)^2 + 45\,a^2b\tan(dx+c) + 12\,a^3}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(abs(tan(d*x + c))) - (411*a^2*b*tan(d*x + c)^5 + 274*b^3*tan(d*x + c)^
5 + 60*a^3*tan(d*x + c)^4 + 360*a*b^2*tan(d*x + c)^4 + 180*a^2*b*tan(d*x +
c)^3 + 30*b^3*tan(d*x + c)^3 + 40*a^3*tan(d*x + c)^2 + 60*a*b^2*tan(d*x + c
)^2 + 45*a^2*b*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

Mupad [B]
time = 3.88, size = 146, normalized size = 0.87

$$\frac{\ln(\tan(c+d\,x))\,(3\,a^2\,b + 2\,b^3)}{d} - \frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)^2\left(\frac{2\,a^3}{3} + a\,b^2\right) + \tan(c+d\,x)^3\left(3\,a^2\,b + \frac{b^3}{2}\right) + \frac{a^3}{5} + \tan(c+d\,x)^4\,(a^3 + 6\,a\,b^2) + \frac{3\,a^2\,b\tan(c+d\,x)}{4}\right)}{d} + \frac{b^3\tan(c+d\,x)^2}{2\,d} + \frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + 2*b^3))/d - (cot(c + d*x)^5*(tan(c + d*x)^2*(
a*b^2 + (2*a^3)/3) + tan(c + d*x)^3*(3*a^2*b + b^3/2) + a^3/5 + tan(c + d*x
)^4*(6*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/4))/d + (b^3*tan(c + d*x)^2)/(
2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=275

$$\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}-\frac{3b^4\cos(c+dx)}{d}$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-10*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)
/d+12*a^2*b^2*cos(d*x+c)/d-3*b^4*cos(d*x+c)/d+1/3*a^4*cos(d*x+c)^3/d-2*a^2*
b^2*cos(d*x+c)^3/d+1/3*b^4*cos(d*x+c)^3/d+6*a^2*b^2*sec(d*x+c)/d-3*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+10*a*b^3*sin(d*x+c)/d-
4/3*a^3*b*sin(d*x+c)^3/d+10/3*a*b^3*sin(d*x+c)^3/d+2*a*b^3*sin(d*x+c)^3*tan
(d*x+c)^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276, 294}

$$\frac{a^4\cos^3(c+dx)}{3d}-\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2b^2\cos^3(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}+\frac{10ab^3\sin^3(c+dx)}{3d}+\frac{10ab^3\sin(c+dx)}{d}+\frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}-\frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^4\cos^3(c+dx)}{3d}-\frac{3b^4\cos(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}-\frac{3b^4\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d +
 (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d
*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*S
ec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d
 - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*
b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin^3(c+dx)+4a^3b\sin^3(c+dx)\tan(c+dx)+6a^2b^2\sin^3(c\right.$$

$$= a^4\int \sin^3(c+dx)\,dx + \left(4a^3b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx + (6a$$

$$= -\frac{a^4\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,d\right.}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{a^4\cos^3(c+dx)}{3d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+d}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} + \frac{a^4\cos^3}{}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} -$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^4\cos(c}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1017 vs. $2(275) = 550$.
time = 6.32, size = 1017, normalized size = 3.70

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b^2*(-36*a^2 + 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(d*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*Cos[c + d
*x]^5*(a + b*Tan[c + d*x])^4)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (
(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^4*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x]
)^4)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^3)*Co
s[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^
4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*Cos[c +
 d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b^4)*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x]
)^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[
```

```
c + d*x])^4) + (Cos[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] - 17*b^4*Sin[(c
 + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] + 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*
b*(5*a^2 - 9*b^2)*Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b*(a^2 - b^2)*Cos[c + d*x]^4*Sin[3*
(c + d*x)]*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
)
```

Maple [A]

time = 0.23, size = 267, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\right.\right.}{}$
default	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\right.\right.}{}$
risch	$-\frac{3\,e^{-i(dx+c)}a^4}{8d}-\frac{11\,e^{-i(dx+c)}b^4}{8d}+\frac{e^{-3i(dx+c)}a^4}{24d}+\frac{e^{-3i(dx+c)}b^4}{24d}+\frac{e^{3i(dx+c)}a^4}{24d}+\frac{e^{3i(dx+c)}b^4}{24d}-\frac{3\,e^{i(dx+c)}a^4}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+4*a^3*b*(-1/3*sin(d*x+c)^3-sin(d*
x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(d*x+c)^7/cos(d*x+c
)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(
d*x+c)))+b^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3
*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]

time = 0.32, size = 218, normalized size = 0.79

$$\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^4-2\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(dx+c)\right)a^3b-6\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^2b^2+\left(4\sin(dx+c)^3-\frac{6\sin(dx+c)}{\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+24\sin(dx+c)\right)ab^3+\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)b^4}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(cos(
```

```
d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^3 -
 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(si
n(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

Fricas [**A**]
time = 0.35, size = 224, normalized size = 0.81

$$\frac{(a^4-6a^2b^2+b^4)\cos(dx+c)^6-3(a^4-12a^2b^2+3b^4)\cos(dx+c)^4+3(2a^3b-5ab^3)\cos(dx+c)^3\log(\sin(dx+c)+1)-3(2a^3b-5ab^3)\cos(dx+c)^3\log(-\sin(dx+c)+1)+b^4+9(2a^2b^2-b^4)\cos(dx+c)^2+2\left(2(a^3b-ab^3)\cos(dx+c)^5+3ab^3\cos(dx+c)-2(4a^3b-7ab^3)\cos(dx+c)^3\right)\sin(dx+c)}{3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)*
cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9*(2
*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 + 3*a*
b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*c
os(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^4\sin^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 7.21, size = 319, normalized size = 1.16

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8a^4-96a^2b^2+32b^4)+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(20ab^3-8a^3b)-\frac{4a^4}{3}-\frac{32b^4}{3}+32a^2b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{32a^4}{3}-64a^2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{20ab^3}{3}-\frac{8a^3b}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}(20ab^3-8a^3b)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{20ab^3}{3}-\frac{8a^3b}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(56ab^3-48a^3b)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(56ab^3-48a^3b)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(20ab^3-8a^3b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^4,x)

[Out] - (tan(c/2 + (d*x)/2)^4*(8*a^4 + 32*b^4 - 96*a^2*b^2) + 4*a^4*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)*(20*a*b^3 - 8*a^3*b) - (4*a^4)/3 - (32*b^4)/3
+ 32*a^2*b^2 - tan(c/2 + (d*x)/2)^6*((32*a^4)/3 - 64*a^2*b^2) + tan(c/2 + (
d*x)/2)^3*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d*x)/2)^11*(20*a*b^3 -
8*a^3*b) - tan(c/2 + (d*x)/2)^9*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d
*x)/2)^5*(56*a*b^3 - 48*a^3*b) + tan(c/2 + (d*x)/2)^7*(56*a*b^3 - 48*a^3*b)
)/(d*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^
12 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(20*a*b^3 - 8*a^3*b))/d
```

3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x-\frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d}+\frac{4ab^3\tan^2(c+dx)}{d}+5$$

```
[Out] 1/2*(a^4-18*a^2*b^2+5*b^4)*x-4*a*b*(a^2-2*b^2)*ln(cos(d*x+c))/d+1/2*b^2*(18
*a^2-5*b^2)*tan(d*x+c)/d+4*a*b^3*tan(d*x+c)^2/d+5/6*b^4*tan(d*x+c)^3/d-1/2*
cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3597, 1659, 815, 649, 209, 266}

$$\frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d}-\frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{1}{2}x\left(a^4-18a^2b^2+5b^4\right)+\frac{4ab^3\tan^2(c+dx)}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^4}{2d}+\frac{5b^4\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^4}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^3(-ab^2}{b^2+x^2}\right.}{} \\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\text{Subst}\left(\int \left(-18a^2b^2\right.\right.}{} \\
&= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} \\
&= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} \\
&= \frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x - \frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{b^2(18a^2}{}
\end{aligned}$$

Mathematica [A]
time = 6.29, size = 263, normalized size = 1.89

$$\frac{b\left(-\frac{(a^4-6a^2b^2+b^4)\text{ArcTan}(\tan(c+dx))}{2b}+2a(a-b)(a+b)\cos^2(c+dx)+\frac{1}{2}\left(4a^3-8ab^2+\frac{a^4-12a^2b^2+3b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(4a^3-8ab^2-\frac{a^4-12a^2b^2+3b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)-\frac{(a^4-6a^2b^2+b^4)\cos(c+dx)\sin(c+dx)}{2b}+2b(3a^2-b^2)\tan(c+dx)+2ab^2\tan^2(c+dx)+\frac{1}{3}b^3\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b*(-1/2*((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 2*a*(a - b)*(a
+ b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 12*a
^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 - ((a^4 - 6
*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*Tan[c
+ d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

Maple [A]
time = 0.20, size = 250, normalized size = 1.80

method	result
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx}{2}\right.\right.}{}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx}{2}\right.\right.}{}$
risch	$\frac{8ib\,a^3c}{d}+\frac{ie^{2i(dx+c)}b^4}{8d}+\frac{a^4x}{2}-9x\,a^2b^2+\frac{5b^4x}{2}+\frac{e^{2i(dx+c)}a^3b}{2d}-\frac{e^{2i(dx+c)}a\,b^3}{2d}-\frac{16ib^3ac}{d}+\frac{3ie^{-2i(dx+c}}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b*(-1/2*sin(d*x+c
)^2-ln(cos(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*si
n(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a*b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+
1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+b^4*(1/3*sin(d*x+c)^7/cos(d
*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8
*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.53, size = 154, normalized size = 1.11

$$\frac{2\,b^4\tan\left(dx+c\right)^3+12\,ab^3\tan\left(dx+c\right)^2+3\left(a^4-18\,a^2b^2+5\,b^4\right)(dx+c)+12\left(a^3b-2\,ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+12\left(3\,a^2b^2-b^4\right)\tan\left(dx+c\right)+\frac{3\left(4\,a^3b-4\,ab^3-\left(a^4-6\,a^2b^2+b^4\right)\tan(dx+c)\right)}{\tan(dx+c)^2+1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^4*tan(d*x + c)^3 + 12*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 18*a^2*b^2 +
 5*b^4)*(d*x + c) + 12*(a^3*b - 2*a*b^3)*log(tan(d*x + c)^2 + 1) + 12*(3*a^
```

```
2*b^2 - b^4)*tan(d*x + c) + 3*(4*a^3*b - 4*a*b^3 - (a^4 - 6*a^2*b^2 + b^4)*
tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [**A**]
time = 0.36, size = 186, normalized size = 1.34

$$\frac{12\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{5}+12\,ab^{3}\cos\left(dx+c\right)-24\left(a^{3}b-2\,ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(-\cos\left(dx+c\right)\right)-3\left(2\,a^{3}b-2\,ab^{3}-\left(a^{4}-18\,a^{2}b^{2}+5\,b^{4}\right)dx\right)\cos\left(dx+c\right)^{3}-\left(3\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}-2\,b^{4}-2\left(18\,a^{2}b^{2}-7\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(12*(a^3*b - a*b^3)*cos(d*x + c)^5 + 12*a*b^3*cos(d*x + c) - 24*(a^3*b
- 2*a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) - 3*(2*a^3*b - 2*a*b^3 - (a^4
- 18*a^2*b^2 + 5*b^4)*d*x)*cos(d*x + c)^3 - (3*(a^4 - 6*a^2*b^2 + b^4)*cos(
d*x + c)^4 - 2*b^4 - 2*(18*a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{4}\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3931 vs. $2(131) = 262$.
time = 2.11, size = 3931, normalized size = 28.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1
5*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 3*a^4*d*x*tan(d*x)^5*tan(c)^3 -
 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 + 15*b^4*d*x*tan(d*x)^5*tan(c)^3 - 9*a^
4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 45*b^4*d*
x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*tan(d*x)^3*tan(c)^5 - 54*a^2*b^2*d*x*tan(
```

d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*x)^3*tan(c)^5 + 6*a^3*b*tan(d*x)^5*tan(c
)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(d
*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2
*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 3*a^4*tan(d*x)^
5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(d*x)^5*tan(c)^4 -
12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c
)^5 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3
*tan(c)^5 + 3*a^4*tan(d*x)^4*tan(c)^5 - 54*a^2*b^2*tan(d*x)^4*tan(c)^5 + 15
*b^4*tan(d*x)^4*tan(c)^5 - 9*a^4*d*x*tan(d*x)^4*tan(c)^2 + 162*a^2*b^2*d*x*
tan(d*x)^4*tan(c)^2 - 45*b^4*d*x*tan(d*x)^4*tan(c)^2 + 12*a^4*d*x*tan(d*x)^
3*tan(c)^3 - 216*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 60*b^4*d*x*tan(d*x)^3*ta
n(c)^3 - 6*a^3*b*tan(d*x)^5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 - 9*a^4
*d*x*tan(d*x)^2*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^2*tan(c)^4 - 45*b^4*d*x
*tan(d*x)^2*tan(c)^4 - 42*a^3*b*tan(d*x)^4*tan(c)^4 + 30*a*b^3*tan(d*x)^4*t
an(c)^4 - 6*a^3*b*tan(d*x)^3*tan(c)^5 + 30*a*b^3*tan(d*x)^3*tan(c)^5 + 36*a
^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2
- 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan
(c)^2 - 36*a^2*b^2*tan(d*x)^5*tan(c)^2 + 10*b^4*tan(d*x)^5*tan(c)^2 - 48*a^
3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 +
96*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(
c)^3 - 12*a^4*tan(d*x)^4*tan(c)^3 + 108*a^2*b^2*tan(d*x)^4*tan(c)^3 - 30*b^
4*tan(d*x)^4*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^2*tan(c)^4 - 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^2*tan(c)^4 - 12*a^4*tan(d*x)^3*tan(c)^4 + 108*a^2*b^
2*tan(d*x)^3*tan(c)^4 - 30*b^4*tan(d*x)^3*tan(c)^4 - 36*a^2*b^2*tan(d*x)^2*
tan(c)^5 + 10*b^4*tan(d*x)^2*tan(c)^5 + 9*a^4*d*x*tan(d*x)^3*tan(c) - 162*a
^2*b^2*d*x*tan(d*x)^3*tan(c) + 45*b^4*d*x*tan(d*x)^3*tan(c) + 12*a*b^3*tan(
d*x)^5*tan(c) - 12*a^4*d*x*tan(d*x)^2*tan(c)^2 + 216*a^2*b^2*d*x*tan(d*x)^2
*tan(c)^2 - 60*b^4*d*x*tan(d*x)^2*tan(c)^2 + 18*a^3*b*tan(d*x)^4*tan(c)^2 -
42*a*b^3*tan(d*x)^4*tan(c)^2 + 9*a^4*d*x*tan(d*x)*tan(c)^3 - 162*a^2*b^2*d
*x*tan(d*x)*tan(c)^3 + 45*b^4*d*x*tan(d*x)*tan(c)^3 + 96*a^3*b*tan(d*x)^3*t

```
an(c)^3 - 48*a*b^3*tan(d*x)^3*tan(c)^3 + 18*a^3*b*tan(d*x)^2*tan(c)^4 - 42*
a*b^3*tan(d*x)^2*tan(c)^4 + 12*a*b^3*tan(d*x)*tan(c)^5 - 2*b^4*tan(d*x)^5 -
 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(
c) + 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*
tan(c) + 72*a^2*b^2*tan(d*x)^4*tan(c) - 30*b^4*tan(d*x)^4*tan(c) + 48*a^3*b
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 96
*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^
2 + 18*a^4*tan(d*x)^3*tan(c)^2 - 108*a^2*b^2*tan(d*x)^3*tan(c)^2 + 10*b^4*t
an(d*x)^3*tan(c)^2 - 36*a^3*b*log(4*(tan(d*x)^4...
```

Mupad [B]
time = 3.79, size = 161, normalized size = 1.16

$$x\left(\frac{a^4}{2}-9\,a^2\,b^2+\frac{5\,b^4}{2}\right)-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(4\,a\,b^3-2\,a^3\,b\right)}{d}-\frac{\cos\left(c+d\,x\right)^2\,\left(\tan(c+d\,x)\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)+2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{\tan(c+d\,x)\,\left(2\,b^4-6\,a^2\,b^2\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

```
[Out] x*(a^4/2 + (5*b^4)/2 - 9*a^2*b^2) - (log(tan(c + d*x)^2 + 1)*(4*a*b^3 - 2*a
^3*b))/d - (cos(c + d*x)^2*(tan(c + d*x)*(a^4/2 + b^4/2 - 3*a^2*b^2) + 2*a*
b^3 - 2*a^3*b))/d + (b^4*tan(c + d*x)^3)/(3*d) - (tan(c + d*x)*(2*b^4 - 6*a
^2*b^2))/d + (2*a*b^3*tan(c + d*x)^2)/d
```

3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=180

$$\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6a^2b^2\cos(c+dx)}{d}-\frac{b^4\cos(c+dx)}{d}+$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-6*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)/
d+6*a^2*b^2*cos(d*x+c)/d-b^4*cos(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d-2*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+6*a*b^3*sin(d*x+c)/d+2
*a*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3598, 2718, 2672, 327, 212, 2670, 14, 294, 276}

$$-\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}+\frac{6ab^3\sin(c+dx)}{d}+\frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}-\frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^4\cos(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}-\frac{2b^4\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^
4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (6*
a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*
d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin[c +
 d*x]*Tan[c + d*x]^2)/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin(c+dx) + 4a^3b\sin(c+dx)\tan(c+dx) + 6a^2b^2\sin(c+dx\right.$$

$$= a^4\int \sin(c+dx)\,dx + \left(4a^3b\right)\int \sin(c+dx)\tan(c+dx)\,dx + \left(6a^2b\right.$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(6a^2b\right.}{}$$

$$= -\frac{a^4\cos(c+dx)}{d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx}{d}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{6a^2b^2\cos(c+dx)}{d} - \frac{b}{}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^4\cos(c+}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(180) = 360.
time = 5.13, size = 383, normalized size = 2.13

$$\frac{72a^2b^2 - 22b^4 - 12(a^4-6a^2b^2+b^4)\cos(c+dx) - 24ab(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 24ab(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{b^3(12a+b)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2b^4\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2(36a^2-11b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - \frac{2b^4\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{b^3(-12a+b)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2b^2(-36a^2+11b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)} - 48ab(a^2-b^2)\sin(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2*
a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3*b
^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

Maple [A]
time = 0.22, size = 217, normalized size = 1.21

method	result
derivativedivides	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a}{}$

default	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a\,b^3}{}$
risch	$\frac{2i\mathrm{e}^{i(dx+c)}a^3b}{d}-\frac{2i\mathrm{e}^{i(dx+c)}a\,b^3}{d}-\frac{\mathrm{e}^{i(dx+c)}a^4}{2d}+\frac{3\,\mathrm{e}^{i(dx+c)}a^2b^2}{d}-\frac{\mathrm{e}^{i(dx+c)}b^4}{2d}-\frac{2i\mathrm{e}^{-i(dx+c)}a^3b}{d}+\frac{2i\mathrm{e}^{-i(dx+c)}a\,b}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*cos(d*x+c)+4*a^3*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*
b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(
d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan
(d*x+c)))+b^4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.32, size = 166, normalized size = 0.92

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-18\,a^2b^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+b^4\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-6\,a^3b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))+3\,a^4\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c)
+ cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x +
c)) + 3*a^4*cos(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 176, normalized size = 0.98

$$-\frac{3\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4-3\left(2\,a^3b-3\,ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^3b-3\,ab^3\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-b^4-6\left(3\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2-6\left(ab^3\cos\left(dx+c\right)-2\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos(
d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 - 6*(a*b^3*c
os(d*x + c) - 2*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^4\sin\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**4,x)

[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 22530 vs. $2(178) = 356$.
time = 8.20, size = 22530, normalized size = 125.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")

```
[Out] -1/3*(6*a^3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2
*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*
tan(1/2*d*x)^8*tan(1/2*c)^8 - 9*a*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 +
2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^
4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 6*a^3*b*log(2*(tan(1/
2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8
 + 9*a*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
 - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^8*tan(1/2*c)^8 + 3*a^4*tan(1/2*d*x)^8*tan(1/2*c)^8 - 36*a^2*b^2*t
an(1/2*d*x)^8*tan(1/2*c)^8 + 8*b^4*tan(1/2*d*x)^8*tan(1/2*c)^8 - 12*a^3*b*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*
tan(1/2*c)^6 + 18*a*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 12*a^3*b*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
```

```
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 18*a*b^3*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*
tan(1/2*c)^6 - 72*a^3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 108*a*b^3*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 72*a^3*b*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7
*tan(1/2*c)^7 - 108*a*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 24*a^3*b*tan(1/2*d*x)^8*tan(1/2*c
)^7 + 36*a*b^3*tan(1/2*d*x)^8*tan(1/2*c)^7 - 12*a^3*b*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 18*a*
b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^8 + 12*a^3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*...
```

Mupad [B]
time = 7.29, size = 268, normalized size = 1.49

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^4-48\,a^2\,b^2+\frac{32\,b^4}{3}\right)+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)-2\,a^4-\frac{16\,b^4}{3}+24\,a^2\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(6\,a^4-24\,a^2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(12\,a\,b^3-8\,a^3\,b\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(20\,a\,b^3-24\,a^3\,b\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(20\,a\,b^3-24\,a^3\,b\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^4,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^2*(6*a^4 + (32*b^4)/3 - 48*a^2*b^2) + 2*a^4*tan(c/2 +
 (d*x)/2)^6 + tan(c/2 + (d*x)/2)*(12*a*b^3 - 8*a^3*b) - 2*a^4 - (16*b^4)/3
+ 24*a^2*b^2 - tan(c/2 + (d*x)/2)^4*(6*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/
2)^7*(12*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(20*a*b^3 - 24*a^3*b) + ta
n(c/2 + (d*x)/2)^5*(20*a*b^3 - 24*a^3*b))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 1)) - (atanh(tan(c/2 + (d*x)/2)
)*(12*a*b^3 - 8*a^3*b))/d
```

3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=118

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{b^4\sec(c}{d}$$

```
[Out] -a^4*arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b^3*arctanh(si
n(d*x+c))/d+6*a^2*b^2*sec(d*x+c)/d-b^4*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3598, 3855, 2686, 8, 2691}

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(c+dx)\sec(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}-\frac{b^4\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a
*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c + d
*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc(c+dx)+4a^3b\sec(c+dx)+6a^2b^2\sec(c+dx)\tan(c+d$$

$$= a^4\int \csc(c+dx)\,dx+\left(4a^3b\right)\int \sec(c+dx)\,dx+\left(6a^2b^2\right)\int \sec(c+$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\sec(c$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab^3\tanh^{-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 352 vs. 2(118) = 236.
time = 4.97, size = 352, normalized size = 2.98

$$\frac{72a^2b^2-10b^4-12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-48a^3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+24ab^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+48a^3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-24ab^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12ab^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+2b^2\left(36a^2-b^2+2b^2\cos(c+dx)+\left(36a^2-5b^2\right)\cos(2(c+dx))\right)\sec^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-\frac{12ab^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a*b
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 - 5*b^2
)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2)/(12*d)
```

Maple [A]
time = 0.28, size = 170, normalized size = 1.44

method	result

derivativedivides	$\frac{a^4 \ln(\csc(dx+c)-\cot(dx+c))+4a^3 b\ln(\sec(dx+c)+\tan(dx+c))+\frac{6a^2b^2}{\cos(dx+c)}+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\cdots}{2}\cdots\right.}{d}$
default	$\frac{a^4 \ln(\csc(dx+c)-\cot(dx+c))+4a^3 b\ln(\sec(dx+c)+\tan(dx+c))+\frac{6a^2b^2}{\cos(dx+c)}+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\cdots}{2}\cdots\right.}{d}$
risch	$-\frac{2b^2\left(-18a^2 e^{5i(dx+c)}+3b^2 e^{5i(dx+c)}+6iab\, e^{5i(dx+c)}-36a^2 e^{3i(dx+c)}+2b^2 e^{3i(dx+c)}-18a^2 e^{i(dx+c)}+3b^2 e^{i(dx+c)}-6iab\, e^{i(\cdots}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(csc(d*x+c)-cot(d*x+c))+4*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*
b^2/cos(d*x+c)+4*a*b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln
(sec(d*x+c)+tan(d*x+c)))+b^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^
4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.48, size = 139, normalized size = 1.18

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^3b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+3\,a^4\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)-\frac{18\,a^2b^2}{\cos(dx+c)}+\frac{\left(3\,\cos(dx+c)^2-1\right)b^4}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 3*a^4*log(cot(d*x + c) + csc(d*x + c)) - 18*a^2*b^2/cos(d*x + c)
+ (3*cos(d*x + c)^2 - 1)*b^4/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 175, normalized size = 1.48

$$-\frac{3\,a^4\cos\left(dx+c\right)^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^4\cos\left(dx+c\right)^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,ab^3\cos\left(dx+c\right)\sin\left(dx+c\right)-6\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)+6\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\,b^4-6\left(6\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^4*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*a^4*cos(d*x + c)
^3*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^3*cos(d*x + c)*sin(d*x + c) - 6*(2
*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^3)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*cos(d*x
 + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (c+dx))^4 \csc (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x), x)
```

Giac [A]
time = 0.99, size = 193, normalized size = 1.64

$$\frac{3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+6\,(2\,a^3b-ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,(2\,a^3b-ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\left(3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,a^2b^2+b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 6*(2*a^3*b - a*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 4*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^
4 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*
b^3*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
/d
```

Mupad [B]
time = 4.14, size = 496, normalized size = 4.20

$$\frac{a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{12\,a^2\,b^2-\frac{4\,b^4}{3}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(4\,b^4-24\,a^2\,b^2\right)+12\,a^2\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,a\,b^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)\,2{}\mathrm{i}+a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)\,2{}\mathrm{i}}{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(64\,a^6\,b^2-64\,a^4\,b^4+16\,a^2\,b^6\right)+16\,a^7\,b-8\,a^5\,b^3+2\,a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)-2\,a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)}\right)\,\left(2\,a^2-b^2\right)\,4{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x),x)
```

```
[Out] (a^4*log(tan(c/2 + (d*x)/2)))/d - (12*a^2*b^2 - (4*b^4)/3 + tan(c/2 + (d*x)
/2)^2*(4*b^4 - 24*a^2*b^2) + 12*a^2*b^2*tan(c/2 + (d*x)/2)^4 + 4*a*b^3*tan(
c/2 + (d*x)/2) - 4*a*b^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 -
 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (a*b*atan((a*b*(2*a^
2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*tan(c/2 + (
d*x)/2)*(2*a^2 - b^2))*2i + a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*ta
n(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))*2i)/(2*tan(c/2
+ (d*x)/2)*(16*a^2*b^6 - 64*a^4*b^4 + 64*a^6*b^2) + 16*a^7*b - 8*a^5*b^3 +
2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*
tan(c/2 + (d*x)/2)*(2*a^2 - b^2)) - 2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b
+ 2*a^4*tan(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))))*(2*
a^2 - b^2)*4i)/d
```

3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=83

$$-\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[Out] -a^4*cot(d*x+c)/d+4*a^3*b*ln(tan(d*x+c))/d+6*a^2*b^2*tan(d*x+c)/d+2*a*b^3*t
an(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 45}

$$-\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*Cot[c + d*x])/d) + (4*a^3*b*Log[Tan[c + d*x]])/d + (6*a^2*b^2*Tan[c
+ d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2a\end{aligned}$$

Mathematica [A]
time = 1.26, size = 162, normalized size = 1.95

$$-\frac{\csc(c+dx)\sec^3(c+dx)\left(4(3a^4+b^4)\cos(2(c+dx))+(3a^4+18a^2b^2-b^4)\cos(4(c+dx))+3(3a^4-6a^2b^2-b^4+8ab(-b^2+a^2\log(\cos(c+dx))-a^2\log(\sin(c+dx)))\sin(2(c+dx))+4a^3b(\log(\cos(c+dx))-\log(\sin(c+dx)))\sin(4(c+dx))\right))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(Csc[c + d*x]*Sec[c + d*x]^3*(4*(3*a^4 + b^4)*Cos[2*(c + d*x)] + (3*a
^4 + 18*a^2*b^2 - b^4)*Cos[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*a*
b*(-b^2 + a^2*Log[Cos[c + d*x]] - a^2*Log[Sin[c + d*x]])*Sin[2*(c + d*x)] +
 4*a^3*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])*Sin[4*(c + d*x)])))/d
```

Maple [A]
time = 0.22, size = 79, normalized size = 0.95

method	result
derivativedivides	$\frac{-a^4\cot(dx+c)+4a^3b\ln(\tan(dx+c))+6a^2b^2\tan(dx+c)+\frac{2a\,b^3}{\cos(dx+c)^2}+\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
default	$\frac{-a^4\cot(dx+c)+4a^3b\ln(\tan(dx+c))+6a^2b^2\tan(dx+c)+\frac{2a\,b^3}{\cos(dx+c)^2}+\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
risch	$-\frac{2i\left(12ia\,b^3\mathrm{e}^{6i(dx+c)}+3a^4\mathrm{e}^{6i(dx+c)}-18a^2b^2\mathrm{e}^{6i(dx+c)}+3b^4\mathrm{e}^{6i(dx+c)}+9a^4\mathrm{e}^{4i(dx+c)}-18a^2b^2\mathrm{e}^{4i(dx+c)}-3b^4\mathrm{e}^{4i(dx+c)}-1\right.}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3\left(\mathrm{e}^{2i(dx+c)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*cot(d*x+c)+4*a^3*b*ln(tan(d*x+c))+6*a^2*b^2*tan(d*x+c)+2*a*b^3/co
s(d*x+c)^2+1/3*b^4*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.31, size = 72, normalized size = 0.87

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\tan\left(dx+c\right)\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\,a^4}{\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(tan(d*x + c
)) + 18*a^2*b^2*tan(d*x + c) - 3*a^4/tan(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 159, normalized size = 1.92

$$-\frac{6\,a^3b\cos\left(dx+c\right)^3\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-6\,a^3b\cos\left(dx+c\right)^3\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-6\,ab^3\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(3\,a^4+18\,a^2b^2-b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(9\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2}{3\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*b*cos(d*x + c)^3*log(cos(d*x + c)^2)*sin(d*x + c) - 6*a^3*b*cos
(d*x + c)^3*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^3*cos(d*x +
 c)*sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*cos(d*x + c)^4 - b^4 - 2*(9*a
^2*b^2 - b^4)*cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^4 \csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.99, size = 86, normalized size = 1.04

$$\frac{b^4\tan(dx+c)^3+6\,ab^3\tan(dx+c)^2+12\,a^3b\log(|\tan(dx+c)|)+18\,a^2b^2\tan(dx+c)-\frac{3\,(4\,a^3b\tan(dx+c)+a^4)}{\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*x
 + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*x
+ c))/d
```

Mupad [B]
time = 3.66, size = 81, normalized size = 0.98

$$\frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{a^4\cot(c+d\,x)}{d}+\frac{6\,a^2\,b^2\tan(c+d\,x)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}+\frac{4\,a^3\,b\,\ln(\tan(c+d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^2,x)
```

```
[Out] (b^4*tan(c + d*x)^3)/(3*d) - (a^4*cot(c + d*x))/d + (6*a^2*b^2*tan(c + d*x)
)/d + (2*a*b^3*tan(c + d*x)^2)/d + (4*a^3*b*log(tan(c + d*x)))/d
```

3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=161

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -1/2*a^4*arctanh(cos(d*x+c))/d-6*a^2*b^2*arctanh(cos(d*x+c))/d+4*a^3*b*arct
anh(sin(d*x+c))/d+2*a*b^3*arctanh(sin(d*x+c))/d-4*a^3*b*csc(d*x+c)/d-1/2*a^
4*cot(d*x+c)*csc(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+2*a
*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {3598, 3853, 3855, 2701, 327, 213, 2702, 2686, 30}

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^4\cot(c+dx)\csc(c+dx)}{2d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(c+dx)\sec(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/2*(a^4*ArcTanh[Cos[c + d*x]])/d - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^3(c+dx)+4a^3b\csc^2(c+dx)\sec(c+dx)+6a^2b^2\csc(c+\right.$$

$$= a^4\int \csc^3(c+dx)\,dx + \left(4a^3b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx + \left(6a\right.$$

$$= -\frac{a^4\cot(c+dx)\csc(c+dx)}{2d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d} + \frac{1}{2}a$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c}{d}$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d} - \frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^3b\tan}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1128 vs. $2(161)=322$.
time = 6.22, size = 1128, normalized size = 7.01

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d*
x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*
(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a
^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4 + 12*a^2
*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Si
n[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
+ d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(c + d
```

```
*x)/2] - b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(36*a^2*b^2*Sin[(c + d*x)/2] + b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x]
)^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^4
)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.30, size = 157, normalized size = 0.98

method	result
derivativedivides	$\frac{a^4\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a^3b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\right.}{d}$
default	$\frac{a^4\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a^3b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\right.}{d}$
risch	$-\frac{i\left(8ib^4e^{7i(dx+c)}-16ib^4e^{5i(dx+c)}+24a^3b\,e^{9i(dx+c)}+12a\,b^3e^{9i(dx+c)}+36ia^2b^2e^{9i(dx+c)}-72ia^2b^2e^{5i(dx+c)}+48a^3b\,e^{7i(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+4*a^3*b
*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*(1/cos(d*x+c)+ln(csc(d
*x+c)-cot(d*x+c)))+4*a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))+1/3*b^4/cos(d*x+c)^3)
```

Maxima [A]
time = 0.31, size = 188, normalized size = 1.17

$$\frac{3\,a^4\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+36\,a^2b^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-24\,a^3b\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{4\,b^4}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 24*a^3*b*(2/sin(d*x + c)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*b^4/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(155) = 310$.
time = 0.42, size = 346, normalized size = 2.15

$$\frac{6\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^4-4\,b^4-4\left(18\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2-3\left(\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^5-\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^5-\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+12\left(\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^5-\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-12\left(\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^5-\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-24\left(ab^3\cos\left(dx+c\right)-\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^5-d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 12*a^2*b^2)*cos(d*x + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*co
s(d*x + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(d*x + c)^5 - (a^4 + 12*a^2*b^2)*co
s(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(d*x +
 c)^5 - (a^4 + 12*a^2*b^2)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 1
2*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^3)*cos(d*x + c)^3)*log
(sin(d*x + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^
3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 24*(a*b^3*cos(d*x + c) - (2*a^3
*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^4 \csc^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**3, x)
```

Giac [A]
time = 1.02, size = 300, normalized size = 1.86

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+12\left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{3\left(6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{16\left(6\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,a^2b^2-b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2*
a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1
/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2 + 16*
(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^4
*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*tan(1
/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 4.07, size = 670, normalized size = 4.16

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{\frac{a^4}{2}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a^4}{2}+48\,a^2b^2+\frac{8b^4}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^4}{2}+48\,a^2b^2+8\,b^4\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{3a^4}{2}+96\,a^2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(16\,a\,b^3-8\,a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(24\,a^3b+16\,a\,b^3\right)+8\,a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+24\,a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{d\left(-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{a^4}{2}+6\,a^2b^2\right)}{d}-\frac{2\,a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\left(2a^2+b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^4+12\,a^2b^2\right)-4\,a\,b^3-8\,a^3b+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2+b^2\right)\right)2\mathrm{i}-a\,b\left(2a^2+b^2\right)\left(4\,a\,b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^4+12\,a^2b^2\right)+8\,a^3b+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2+b^2\right)\right)2\mathrm{i}}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(64\,a^6b^2+64\,a^4b^4+16\,a^2b^6\right)+8\,a^7b+48\,a^3b^5+100\,a^5b^3-2\,a\,b\left(2a^2+b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^4+12\,a^2b^2\right)-4\,a\,b^3-8\,a^3b+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2+b^2\right)\right)-2\,a\,b\left(2a^2+b^2\right)\left(4\,a\,b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^4+12\,a^2b^2\right)+8\,a^3b+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2+b^2\right)\right)}\right)\left(2a^2+b^2\right)4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^3,x)

[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4/2 - tan(c/2 + (d*x)/2)^2*((3*a^4)/2
 + (8*b^4)/3 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(a^4/2 + 8*b^4 + 48*a^2*b
^2) + tan(c/2 + (d*x)/2)^4*((3*a^4)/2 + 96*a^2*b^2) + tan(c/2 + (d*x)/2)^7*
(16*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(16*a*b^3 + 24*a^3*b) + 8*a^3*b
*tan(c/2 + (d*x)/2) + 24*a^3*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/
2)^2 - 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x
)/2)^8)) + (log(tan(c/2 + (d*x)/2))*(a^4/2 + 6*a^2*b^2))/d - (2*a^3*b*tan(c
/2 + (d*x)/2))/d - (a*b*atan((a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 +
12*a^2*b^2) - 4*a*b^3 - 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*
2i - a*b*(2*a^2 + b^2)*(4*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8
*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*2i)/(2*tan(c/2 + (d*x)/2)
*(16*a^2*b^6 + 64*a^4*b^4 + 64*a^6*b^2) + 8*a^7*b + 48*a^3*b^5 + 100*a^5*b^
3 - 2*a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) - 4*a*b^3 -
8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)) - 2*a*b*(2*a^2 + b^2)*(4
*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8*a^3*b + 12*a*b*tan(c/2 +
 (d*x)/2)*(2*a^2 + b^2))))*(2*a^2 + b^2)*4i)/d
```

3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=137

$$-\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d}+\frac{b^2(6a^2+b}{}$$

```
[Out] -a^2*(a^2+6*b^2)*cot(d*x+c)/d-2*a^3*b*cot(d*x+c)^2/d-1/3*a^4*cot(d*x+c)^3/d
+4*a*b*(a^2+b^2)*ln(tan(d*x+c))/d+b^2*(6*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$-\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^3b\cot^2(c+dx)}{d}+\frac{b^2(6a^2+b^2)\tan(c+dx)}{d}-\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (2*a^3*b*Cot[c + d*x]^2)/d - (a^4*C
ot[c + d*x]^3)/(3*d) + (4*a*b*(a^2 + b^2)*Log[Tan[c + d*x]])/d + (b^2*(6*a^
2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3
)/(3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^4(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(6a^2\left(1+\frac{b^2}{6a^2}\right)+\frac{a^4b^2}{x^4}+\frac{4a^3b^2}{x^3}+\frac{a^4+6a^2b^2}{x^2}+\frac{4a(a^2+b^2)}{x}+4ax\right.}{d}$$

$$= -\frac{a^2(a^2+6b^2)\cot(c+dx)}{d} - \frac{2a^3b\cot^2(c+dx)}{d} - \frac{a^4\cot^3(c+dx)}{3d} +$$

Mathematica [A]

time = 3.78, size = 188, normalized size = 1.37

$$-\frac{(b+a\cot(c+dx))^4\sin(c+dx)\left(\cos(c+dx)\left(-6ab^3+6a^3b\cot^2(c+dx)+a^4\cot^3(c+dx)\right)+2a\cos^3(c+dx)\left(a(a^2+9b^2)\cot(c+dx)+6b(a^2+b^2)\left(\log(\cos(c+dx))-\log(\sin(c+dx))\right)\right)-b^4\sin(c+dx)-2b^2(9a^2+b^2)\cos^2(c+dx)\sin(c+dx)\right)\tan^3(c+dx)}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/3*((b + a*Cot[c + d*x])^4*Sin[c + d*x]*(Cos[c + d*x]*(-6*a*b^3 + 6*a^3*b
*Cot[c + d*x]^2 + a^4*Cot[c + d*x]^3) + 2*a*Cos[c + d*x]^3*(a*(a^2 + 9*b^2)
*Cot[c + d*x] + 6*b*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) -
b^4*Sin[c + d*x] - 2*b^2*(9*a^2 + b^2)*Cos[c + d*x]^2*Sin[c + d*x])*Tan[c +
 d*x]^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]

time = 0.28, size = 133, normalized size = 0.97

method	result
derivativedivides	$\frac{a^4\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+4a^3b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+4a}{d}$
default	$\frac{a^4\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+4a^3b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+4a}{d}$
risch	$\frac{8a^3b\,\mathrm{e}^{10i(dx+c)}+8a\,b^3\mathrm{e}^{10i(dx+c)}-24ia^2b^2+4ia^4\mathrm{e}^{8i(dx+c)}-\frac{4ib^4}{3}+16a^3b\,\mathrm{e}^{8i(dx+c)}-16a\,b^3\mathrm{e}^{8i(dx+c)}+4ib^4\mathrm{e}^{8i(dx+c)}-\frac{32ib^4\mathrm{e}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+4*a^3*b*(-1/2/sin(d*x+c)^2+ln(t
an(d*x+c)))+6*a^2*b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+4*a*b^3*(1/2/c
os(d*x+c)^2+ln(tan(d*x+c)))-b^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 120, normalized size = 0.88

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\left(a^3b+ab^3\right)\log\left(\tan\left(dx+c\right)\right)+3\left(6\,a^2b^2+b^4\right)\tan\left(dx+c\right)-\frac{6\,a^3b\tan(dx+c)+a^4+3\left(a^4+6\,a^2b^2\right)\tan(dx+c)^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*log(t
an(d*x + c)) + 3*(6*a^2*b^2 + b^4)*tan(d*x + c) - (6*a^3*b*tan(d*x + c) + a
^4 + 3*(a^4 + 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(133) = 266$.
time = 0.37, size = 267, normalized size = 1.95

$$-\frac{2\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+18\,a^2b^2\cos\left(dx+c\right)^2-3\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+6\left(\left(a^3b+ab^3\right)\cos\left(dx+c\right)^5-\left(a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-6\left(\left(a^3b+ab^3\right)\cos\left(dx+c\right)^5-\left(a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+6\left(ab^3\cos\left(dx+c\right)-\left(a^3b+ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^5-d\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^2
 - 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*cos
(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*x +
 c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*l
og(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (a^3*b
 + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*x + c)
^3)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^4\csc^4\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**4, x)
```

Giac [A]
time = 1.04, size = 161, normalized size = 1.18

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+18\,a^2b^2\tan\left(dx+c\right)+3\,b^4\tan\left(dx+c\right)+12\left(a^3b+ab^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{22\,a^3b\tan(dx+c)^3+22\,ab^3\tan(dx+c)^3+3\,a^4\tan(dx+c)^2+18\,a^2b^2\tan(dx+c)^2+6\,a^3b\tan(dx+c)+a^4}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
+ 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c))) - (22*a^3*
b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a^2*
b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3)/d
```

Mupad [B]
time = 3.76, size = 132, normalized size = 0.96

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(\tan(c+d\,x)^2\,\left(a^4+6\,a^2\,b^2\right)+\frac{a^4}{3}+2\,a^3\,b\tan(c+d\,x)\right)}{d}+\frac{\tan(c+d\,x)\,\left(6\,a^2\,b^2+b^4\right)}{d}+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(4*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^3*(tan(c + d*x)^2
*(a^4 + 6*a^2*b^2) + a^4/3 + 2*a^3*b*tan(c + d*x)))/d + (tan(c + d*x)*(b^4
+ 6*a^2*b^2))/d + (b^4*tan(c + d*x)^3)/(3*d) + (2*a*b^3*tan(c + d*x)^2)/d
```

3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=274

$$-\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -3/8*a^4*arctanh(cos(d*x+c))/d-9*a^2*b^2*arctanh(cos(d*x+c))/d-b^4*arctanh(
cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+6*a*b^3*arctanh(sin(d*x+c))/d-4
*a^3*b*csc(d*x+c)/d-6*a*b^3*csc(d*x+c)/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d-4/
3*a^3*b*csc(d*x+c)^3/d-1/4*a^4*cot(d*x+c)*csc(d*x+c)^3/d+9*a^2*b^2*sec(d*x+
c)/d+b^4*sec(d*x+c)/d-3*a^2*b^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b^3*csc(d*x+c
)*sec(d*x+c)^2/d+1/3*b^4*sec(d*x+c)^3/d
```

Rubi [A]

time = 0.19, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$-\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{9a^2b^2\sec(c+dx)}{d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}-\frac{6ab^3\csc(c+dx)}{d}+\frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}+\frac{b^4\sec(c+dx)}{d}-\frac{b^4\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^5(c+dx)+4a^3b\csc^4(c+dx)\sec(c+dx)+6a^2b^2\csc^3(c+\right.$$

$$= a^4\int \csc^5(c+dx)\,dx+\left(4a^3b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(6a\right.$$

$$= -\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)\int \csc^3(c+dx)\,dx-\frac{\left(4a^3b\right.}{}$$

$$= -\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2}{}$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{6ab^3\csc(c+dx)}{d}$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^4\tanh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1491 vs. $2(274)=548$.
time = 6.30, size = 1491, normalized size = 5.44

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + ((-7*a^3*b*Cos[(c + d*x)/2] - 6*a*b^3*Cos[(c
+ d*x)/2])*Cos[c + d*x]^4*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) - (3*(a^4 + 8*a^2*b^2)*Cos[c + d*x]^4*Csc
[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^4) - (a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d
*x]^4*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((-3*a^4 - 72*a^2*b^2 - 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c
 + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) +
((3*a^4 + 72*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 3
*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(a^4 + 8*a^2*b^2)
```

```
*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(a + b*
Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 +
b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
 + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]
^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*Sec[(c + d*x)/2]*(-7*a
^3*b*Sin[(c + d*x)/2] - 6*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(
3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] - 7*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos
[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] + 7*b^4*Sin[(c + d*x)/2])*(a + b*T
an[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4) - (a^3*b*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.33, size = 241, normalized size = 0.88

method	result
derivativedivides	$\frac{a^4\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+4a^3b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+\right.}{}$
default	$\frac{a^4\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+4a^3b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+\right.}{}$
risch	$\frac{-216a^2b^2e^{9i(dx+c)}+288a^2b^2e^{7i(dx+c)}-216a^2b^2e^{5i(dx+c)}-144a^2b^2e^{3i(dx+c)}+216a^2b^2e^{i(dx+c)}-16b^4e^{11i(dx+c)}-6a^4e^{11i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-c
ot(d*x+c)))+4*a^3*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c
)))+6*a^2*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c
)-cot(d*x+c)))+4*a*b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(s
ec(d*x+c)+tan(d*x+c)))+b^4*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot
(d*x+c))))
```

Maxima [A]
time = 0.34, size = 304, normalized size = 1.11

$$\frac{3a^4\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+72a^2b^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-48ab^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+8b^4\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-32a^3b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72*a
^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c)^2
- 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x + c)^2 +
 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(264) = 528$.
time = 0.47, size = 547, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b^
2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2 -
 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8
*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(1/2
*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(
3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*c
os(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3)*cos(d
*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(
d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7
- 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3
)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + 3*a*b
^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*co
s(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^4 \csc^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**5, x)
```

Giac [A]
time = 1.05, size = 479, normalized size = 1.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 480*a^3*b
*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) + 384*(2*a^3*b + 3*a
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 384*(2*a^3*b + 3*a*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 24*(3*a^4 + 72*a^2*b^2 + 8*b^4)*log(abs(tan(1/
2*d*x + 1/2*c))) + 256*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*
d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 + 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c) - 9*a^
2*b^2 - 2*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (150*a^4*tan(1/2*d*x + 1/2*
c)^4 + 3600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4
 + 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 24
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 32*a^3*b
*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 4.17, size = 857, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^5,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^4)/(64*d) - (atan(-((6*a*b^3 + 4*a^3*b)*(12*a*b^3 +
 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((
3*a^4)/4 + 2*b^4 + 18*a^2*b^2))*1i + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*
b + 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/
4 + 2*b^4 + 18*a^2*b^2))*1i)/(2*tan(c/2 + (d*x)/2)*(144*a^2*b^6 + 192*a^4*b
^4 + 64*a^6*b^2) + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b - 6*tan(c/2 + (d
*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2
*b^2)) - (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(6*
a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2*b^2)) + 2
4*a*b^7 + 6*a^7*b + 232*a^3*b^5 + 153*a^5*b^3))*(a*b^3*12i + a^3*b*8i))/d -
 (tan(c/2 + (d*x)/2)*(2*a^3*b + (a*b*(a^2 + 4*b^2))/2))/d + (log(tan(c/2 +
(d*x)/2))*((3*a^4)/8 + b^4 + 9*a^2*b^2))/d + (tan(c/2 + (d*x)/2)^2*(a^4/8 +
 (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)^6*((23*a^4)/4 + 64*b^4 + 420*a^2*b
^2) - tan(c/2 + (d*x)/2)^4*((21*a^4)/4 + (128*b^4)/3 + 228*a^2*b^2) - tan(c
/2 + (d*x)/2)^8*(2*a^4 + 64*b^4 + 204*a^2*b^2) + a^4/4 + tan(c/2 + (d*x)/2)
```

```
^2*((5*a^4)/4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(32*a*b^3 + 32*a^3*b) +
tan(c/2 + (d*x)/2)^9*(32*a*b^3 - 40*a^3*b) - tan(c/2 + (d*x)/2)^5*(160*a*b^
3 + 112*a^3*b) + tan(c/2 + (d*x)/2)^7*(96*a*b^3 + (352*a^3*b)/3) + (8*a^3*b
*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 - 48*tan(c/2 + (d*x)/2)
^6 + 48*tan(c/2 + (d*x)/2)^8 - 16*tan(c/2 + (d*x)/2)^10)) - (a^3*b*tan(c/2
+ (d*x)/2)^3)/(6*d)
```

3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=194

$$-\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d}-\frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d}-\frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{a^3b\cot^4(c+dx)}{d}-$$

```
[Out] -(a^4+12*a^2*b^2+b^4)*cot(d*x+c)/d-2*a*b*(2*a^2+b^2)*cot(d*x+c)^2/d-2/3*a^2
*(a^2+3*b^2)*cot(d*x+c)^3/d-a^3*b*cot(d*x+c)^4/d-1/5*a^4*cot(d*x+c)^5/d+4*a
*b*(a^2+2*b^2)*ln(tan(d*x+c))/d+2*b^2*(3*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 962}

$$-\frac{a^4\cot^5(c+dx)}{5d}-\frac{a^3b\cot^4(c+dx)}{d}+\frac{2b^2(3a^2+b^2)\tan(c+dx)}{d}-\frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d}+\frac{4ab(a^2+2b^2)\log(\tan(c+dx))}{d}-\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c +
 d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*
x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x
]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d +
 (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^4\left(b^2+x^2\right)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(2(3a^2+b^2)+\frac{a^4b^4}{x^6}+\frac{4a^3b^4}{x^5}+\frac{2a^2b^2\left(a^2+3b^2\right)}{x^4}+\frac{4ab^2\left(2a^2+b^2\right)}{x^3}\right.}{d}$$

$$= -\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d} - \frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d} - $$

Mathematica [A]
time = 4.33, size = 233, normalized size = 1.20

$$-\frac{\left(15a^3b\cot^4(c+dx)+3a^4\cot^5(c+dx)+2a\cos^2(c+dx)\left(-15b^3+15b(a^2+b^2)\cot^2(c+dx)+a(2a^2+15b^2)\cot^3(c+dx)\right)+\cos^4(c+dx)\left((8a^4+150a^2b^2+15b^4)\cot(c+dx)+60ab(a^2+2b^2)\left(\log(\cos(c+dx))-\log(\sin(c+dx))\right)\right)-5b^2(18a^2+5b^2)\cos^3(c+dx)\sin(c+dx)-\frac{5}{2}b^4\sin(2(c+dx))\right)(a+b\tan(c+dx))^4}{15d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/15*((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^2
*(-15*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c + d*
x]^3) + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60*a*
b*(a^2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^2 +
5*b^2)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)])/2)*(a + b*Tan
[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.29, size = 218, normalized size = 1.12

method	result
derivativedivides	$\frac{a^4\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+4a^3b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(-\frac{}{3\sin}\right.}{}$
default	$\frac{a^4\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+4a^3b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(-\frac{}{3\sin}\right.}{}$
risch	$\frac{16ab^3e^{10i(dx+c)}-32ia^2b^2-\frac{80ia^4e^{8i(dx+c)}}{3}+\frac{112ib^4e^{8i(dx+c)}}{3}-\frac{128ib^4e^{6i(dx+c)}}{3}+\frac{32ib^4e^{4i(dx+c)}}{3}-\frac{256ia^4e^{6i(dx+c)}}{15}+16a^3b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+4*a^3*b*(-1/
4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+6*a^2*b^2*(-1/3/sin(d*x+c)^
3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+4*a*b^3*(1/2/sin(d*x
```

```
+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c)))+b^4*(1/3/sin(d*x+c)/cos
(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c)))
```

Maxima [A]
time = 0.33, size = 171, normalized size = 0.88

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+60\left(a^3b+2\,ab^3\right)\log\left(\tan\left(dx+c\right)\right)+30\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)-\frac{15\,a^3b\tan(dx+c)+15\,(a^4+12\,a^2b^2+b^4)\tan(dx+c)^4+3\,a^4+30\,(2\,a^3b+ab^3)\tan(dx+c)^3+10\,(a^4+3\,a^2b^2)\tan(dx+c)^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3)
*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*x
+ c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b + a
*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(188) = 376.
time = 0.37, size = 386, normalized size = 1.99

$$-\frac{8\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^8-20\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6+15\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^4-5\,b^4-10\left(9\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+30\left(\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^7-2\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^5+\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-30\left(\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^7-2\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^5+\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^3\right)\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-15\left(2\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^5+2\,ab^3\cos\left(dx+c\right)-3\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^7-2\,d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2 +
 5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5*b
^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^
3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x + c)^7
- 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^3)*lo
g(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*cos(d*x
 + c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*
x + c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)*sin(d*x
 + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.07, size = 235, normalized size = 1.21

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+90\,a^2b^2\tan\left(dx+c\right)+30\,b^4\tan\left(dx+c\right)+60\,\left(a^3b+2\,ab^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{137\,a^3b\tan(dx+c)^5+274\,ab^3\tan(dx+c)^5+15\,a^4\tan(dx+c)^4+180\,a^2b^2\tan(dx+c)^4+15\,b^4\tan(dx+c)^4+60\,a^3b\tan(dx+c)^3+30\,ab^3\tan(dx+c)^3+10\,a^4\tan(dx+c)^2+30\,a^2b^2\tan(dx+c)^2+15\,a^3b\tan(dx+c)+3\,a^4}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x +
 c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c))) - (
137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c)^4
 + 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*x +
c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*tan(d*x
 + c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

Mupad [B]
time = 3.83, size = 181, normalized size = 0.93

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,a^3\,b+8\,a\,b^3\right)}{d}-\frac{\cot(c+d\,x)^5\,\left(\tan(c+d\,x)^2\,\left(\frac{2\,a^4}{3}+2\,a^2\,b^2\right)+\tan(c+d\,x)^3\,\left(4\,a^3\,b+2\,a\,b^3\right)+\frac{a^4}{5}+\tan(c+d\,x)^4\,\left(a^4+12\,a^2\,b^2+b^4\right)+a^3\,b\tan(c+d\,x)\right)}{d}+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(6\,a^2\,b^2+2\,b^4\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(8*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^5*(tan(c + d*x)^2
*((2*a^4)/3 + 2*a^2*b^2) + tan(c + d*x)^3*(2*a*b^3 + 4*a^3*b) + a^4/5 + tan
(c + d*x)^4*(a^4 + b^4 + 12*a^2*b^2) + a^3*b*tan(c + d*x)))/d + (b^4*tan(c
+ d*x)^3)/(3*d) + (tan(c + d*x)*(2*b^4 + 6*a^2*b^2))/d + (2*a*b^3*tan(c + d
*x)^2)/d
```

3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=402

$$-\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{5b^4\tanh^{-1}(\cos(c+dx))}{2d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -5/16*a^4*arctanh(cos(d*x+c))/d-45/4*a^2*b^2*arctanh(cos(d*x+c))/d-5/2*b^4*
arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+10*a*b^3*arctanh(sin(d*
x+c))/d-4*a^3*b*csc(d*x+c)/d-10*a*b^3*csc(d*x+c)/d-5/16*a^4*cot(d*x+c)*csc(
d*x+c)/d-4/3*a^3*b*csc(d*x+c)^3/d-10/3*a*b^3*csc(d*x+c)^3/d-5/24*a^4*cot(d*
x+c)*csc(d*x+c)^3/d-4/5*a^3*b*csc(d*x+c)^5/d-1/6*a^4*cot(d*x+c)*csc(d*x+c)^
5/d+45/4*a^2*b^2*sec(d*x+c)/d+5/2*b^4*sec(d*x+c)/d-15/4*a^2*b^2*csc(d*x+c)^
2*sec(d*x+c)/d-3/2*a^2*b^2*csc(d*x+c)^4*sec(d*x+c)/d+2*a*b^3*csc(d*x+c)^3*s
ec(d*x+c)^2/d+5/6*b^4*sec(d*x+c)^3/d-1/2*b^4*csc(d*x+c)^2*sec(d*x+c)^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$-\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}-\frac{4a^3b\csc^5(c+dx)}{5d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{45a^2b^2\sec(c+dx)}{4d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}-\frac{10ab^3\csc^3(c+dx)}{3d}-\frac{10ab^3\csc(c+dx)}{d}+\frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d}+\frac{5b^4\sec^3(c+dx)}{6d}+\frac{5b^4\sec(c+dx)}{2d}-\frac{5b^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/
(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]
])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*
a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b
*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c +
d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c
 + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^
2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]
^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/
(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^7(c+dx)+4a^3b\csc^6(c+dx)\sec(c+dx)+6a^2b^2\csc^5(c+\right.$$

$$= a^4\int \csc^7(c+dx)\,dx + \left(4a^3b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx + \left(6a^2\right.$$

$$= -\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}\left(5a^4\right)\int \csc^5(c+dx)\,dx - \frac{\left(4a^3b\right)}{}$$

$$= -\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{3a^2b}{}$$

$$= -\frac{4a^3b\csc(c+dx)}{d} - \frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} - \frac{4a^3b\csc^3(c+dx}{3d}$$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d} + \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c}{d}$$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d} - \frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d} - \frac{5b^4\tan}{}$$

Mathematica [A]
time = 6.28, size = 660, normalized size = 1.64

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*(a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*(a + b*
Tan[c + d*x])^4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b +
 5*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (5*(a^4 + 36*a^2*b
^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(1
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cot[c + d*x]*Csc[c + d*x]^5*(-2545*a
^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*Cos[2*(c + d*x)] - 7200*a^2*b^2*Cos[
2*(c + d*x)] - 6720*b^4*Cos[2*(c + d*x)] + 60*a^4*Cos[4*(c + d*x)] + 2160*a
```

```
^2*b^2*Cos[4*(c + d*x)] + 480*b^4*Cos[4*(c + d*x)] + 200*a^4*Cos[6*(c + d*x
)] + 7200*a^2*b^2*Cos[6*(c + d*x)] + 1600*b^4*Cos[6*(c + d*x)] - 75*a^4*Cos
[8*(c + d*x)] - 2700*a^2*b^2*Cos[8*(c + d*x)] - 600*b^4*Cos[8*(c + d*x)] -
15744*a^3*b*Sin[2*(c + d*x)] - 8640*a*b^3*Sin[2*(c + d*x)] - 1152*a^3*b*Sin
[4*(c + d*x)] - 2880*a*b^3*Sin[4*(c + d*x)] + 3200*a^3*b*Sin[6*(c + d*x)] +
 8000*a*b^3*Sin[6*(c + d*x)] - 960*a^3*b*Sin[8*(c + d*x)] - 2400*a*b^3*Sin[
8*(c + d*x)])*(a + b*Tan[c + d*x])^4)/(30720*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4)
```

Maple [A]
time = 0.43, size = 327, normalized size = 0.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+
5/16*ln(csc(d*x+c)-cot(d*x+c)))+4*a^3*b*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3
-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*(-1/4/sin(d*x+c)^4/cos(d
*x+c)-5/8/sin(d*x+c)^2/cos(d*x+c)+15/8/cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*
x+c)))+4*a*b^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-
5/2/sin(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+c)))+b^4*(1/3/sin(d*x+c)^2/cos(d*x
+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+
c))))
```

Maxima [A]
time = 0.34, size = 387, normalized size = 0.96

$$\frac{5\,a^4\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+40\,b^4\left(\frac{2\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+180\,a^2b^2\left(\frac{2\left(15\,\cos(dx+c)^4-25\,\cos(dx+c)^2+8\right)}{\cos(dx+c)^5-2\,\cos(dx+c)^3+\cos(dx+c)}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-160\,ab^3\left(\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-64\,a^3b\left(\frac{2\left(15\,\sin(dx+c)^4+5\,\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(5*a^4*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*x
+ c) + 1) + 15*log(cos(d*x + c) - 1)) + 40*b^4*(2*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1)) + 180*a^2*b^2*(2*(15*cos(d*x + c)^4 - 25*c
os(d*x + c)^2 + 8)/(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 160*a*b^3*(2*(15*sin(d*
x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 - sin(d*x + c)^3) - 15*lo
g(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 64*a^3*b*(2*(15*sin(d*x +
 c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.49, size = 697, normalized size = 1.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(150*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^8 - 400*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^6 + 330*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 -
 160*b^4 - 480*(6*a^2*b^2 + b^4)*cos(d*x + c)^2 - 75*((a^4 + 36*a^2*b^2 + 8
*b^4)*cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4
 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 75*((a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*
x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(
-1/2*cos(d*x + c) + 1/2) + 480*((2*a^3*b + 5*a*b^3)*cos(d*x + c)^9 - 3*(2*a
^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - (2*
a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 480*((2*a^3*b + 5*
a*b^3)*cos(d*x + c)^9 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b +
 5*a*b^3)*cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(-sin(d*x
 + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 - 35*(2*a^3*b + 5*a*
b^3)*cos(d*x + c)^5 - 15*a*b^3*cos(d*x + c) + 23*(2*a^3*b + 5*a*b^3)*cos(d*
x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9 - 3*d*cos(d*x + c)^7 + 3*d*cos(d*
x + c)^5 - d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.22, size = 647, normalized size = 1.61

$$\frac{5a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-48a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+180a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-560a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-320ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+225a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2880a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+240b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5280a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8640ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3840\left(2a^3b+5ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3840\left(2a^3b+5ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+600\left(a^4+36a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+\frac{1280\left(6ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-9b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+36a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-18a^2b^2-7b^4\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}-\frac{1470a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+52920a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+11760b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+5280a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+8640ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+225a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2880a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+560a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+320ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+45a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+180a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+48a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5a^4}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 45
*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^4*tan(1
/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1/2*d
*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) - 8640*a*b^3*tan(1/2*d*x +
1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3840
```

```
*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 + 36*a^2
*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*tan(1/2*d*x +
1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 9*b^4*tan(1/2*d*x + 1/2*c)^4
 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^4*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3 - (1470*a^4*tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*tan(1/2*d*x + 1/2*c)^
6 + 11760*b^4*tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
8640*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 225*a^4*tan(1/2*d*x + 1/2*c)^4 + 2880*a
^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^4*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b*
tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^4*tan(1/2*
d*x + 1/2*c)^2 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a^3*b*tan(1/2*d*x
+ 1/2*c) + 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 4.03, size = 990, normalized size = 2.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^7,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^6)/(384*d) - (atan(-((10*a*b^3 + 4*a^3*b)*(20*a*b^3
 + 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)
*((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i + (10*a*b^3 + 4*a^3*b)*(20*a*b^3
+ 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*
((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i)/(2*tan(c/2 + (d*x)/2)*(400*a^2*b^
6 + 320*a^4*b^4 + 64*a^6*b^2) + (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b -
6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 +
 5*b^4 + (45*a^2*b^2)/2)) - (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b + 6*ta
n(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 + 5*b
^4 + (45*a^2*b^2)/2)) + 100*a*b^7 + 5*a^7*b + 490*a^3*b^5 + (385*a^5*b^3)/2
))*(a*b^3*20i + a^3*b*8i))/d + (tan(c/2 + (d*x)/2)^4*((a^2*(a^2 + 12*b^2))/
128 + a^4/64))/d + (tan(c/2 + (d*x)/2)^2*((a^2*(a^2 + 12*b^2))/16 + (7*a^4)
/128 + b^4/8 + (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)*((9*a*b^3)/2 + (11*a
^3*b)/4))/d - (tan(c/2 + (d*x)/2)^4*((7*a^4)/2 + 8*b^4 + 78*a^2*b^2) - tan(
c/2 + (d*x)/2)^10*((15*a^4)/2 + 392*b^4 + 864*a^2*b^2) - tan(c/2 + (d*x)/2)
^6*((109*a^4)/6 + (968*b^4)/3 + 1038*a^2*b^2) + tan(c/2 + (d*x)/2)^8*(21*a^
4 + 536*b^4 + 1818*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(a^4 + 6*a^2*b^2) + a^4/
6 - tan(c/2 + (d*x)/2)^11*(32*a*b^3 + 176*a^3*b) + tan(c/2 + (d*x)/2)^3*((3
2*a*b^3)/3 + (208*a^3*b)/15) + tan(c/2 + (d*x)/2)^5*(256*a*b^3 + (624*a^3*b
)/5) - tan(c/2 + (d*x)/2)^7*(1088*a*b^3 + (2368*a^3*b)/5) + tan(c/2 + (d*x)
/2)^9*((2560*a*b^3)/3 + (1528*a^3*b)/3) + (8*a^3*b*tan(c/2 + (d*x)/2))/5)/(
d*(64*tan(c/2 + (d*x)/2)^6 - 192*tan(c/2 + (d*x)/2)^8 + 192*tan(c/2 + (d*x)
/2)^10 - 64*tan(c/2 + (d*x)/2)^12)) - (tan(c/2 + (d*x)/2)^3*((a*b^3)/6 + (7
*a^3*b)/24))/d + (log(tan(c/2 + (d*x)/2))*((5*a^4)/16 + (5*b^4)/2 + (45*a^2
*b^2)/4))/d - (a^3*b*tan(c/2 + (d*x)/2)^5)/(40*d)
```

3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=274

$$\frac{a^5 b \tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2} d}+\frac{a^3 b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d}+\frac{a b^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right) d}-\frac{a b^2\cos^3(c+dx)}{3\left(a^2+b^2\right)^2 d}+\frac{2a\,c}{3}$$

```
[Out] a^5*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)/
d+a^3*b^2*cos(d*x+c)/(a^2+b^2)^3/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x
+c)/(a^2+b^2)/d-1/3*a*b^2*cos(d*x+c)^3/(a^2+b^2)^2/d+2/3*a*cos(d*x+c)^3/(a^
2+b^2)/d-1/5*a*cos(d*x+c)^5/(a^2+b^2)/d+a^4*b*sin(d*x+c)/(a^2+b^2)^3/d+1/3*
a^2*b*sin(d*x+c)^3/(a^2+b^2)^2/d+1/5*b*sin(d*x+c)^5/(a^2+b^2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3599, 3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{a^2 b\sin^3(c+dx)}{3d\left(a^2+b^2\right)^2}-\frac{a\cos^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{2a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a b^2\cos^3(c+dx)}{3d\left(a^2+b^2\right)^2}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{a b^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^5 b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}+\frac{a^4 b\sin(c+dx)}{d\left(a^2+b^2\right)^3}+\frac{a^3 b^2\cos(c+dx)}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d
*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c +
d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*C
os[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) +
 (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 +
b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
```

```
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)\sin^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{a\int \sin^5(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^4(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}$$
$$= \frac{a^2b\sin^3(c+dx)}{3\left(a^2+b^2\right)^2 d} - \frac{\left(a^3b\right)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin^3(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\mathrm{Su}}{}$$
$$= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{2a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} - \frac{a\cos^5(c+dx)}{5\left(a^2+b^2\right)d} + \frac{a^4b\sin(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{a^2b\sin^3(c}{3\left(a^2+\right.}$$
$$= \frac{a^3b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} - \frac{ab^2\cos^3(c+dx)}{3\left(a^2+b^2\right)^2 d} + \frac{2a\cos^3(}{3\left(a^2+\right.}$$
$$= \frac{a^5b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2} d} + \frac{a^3b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c}{\left(a^2+\right.}$$

Mathematica [A]
time = 3.37, size = 289, normalized size = 1.05

$$\frac{-480a^5b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\left(-30a\left(5a^4-4a^2b^2-b^4\right)\cos(c+dx)+5a\left(5a^4+6a^2b^2+b^4\right)\cos(3(c+dx))-3a^5\cos(5(c+dx))-6a^3b^2\cos(5(c+dx))-3ab^4\cos(5(c+dx))+330a^4b\sin(c+dx)+120a^2b^3\sin(c+dx)+30b^5\sin(c+dx)-35a^4b\sin(3(c+dx))-50a^2b^3\sin(3(c+dx))-15b^5\sin(3(c+dx))+3a^4b\sin(5(c+dx))+6a^2b^3\sin(5(c+dx))+3b^5\sin(5(c+dx))\right)}{240\left(a^2+b^2\right)^{7/2} d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
 b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*b^
2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c + d
*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*Sin[
c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin
[3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] + 6*a^2*
b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/2)*d)
```

Maple [A]
time = 0.40, size = 358, normalized size = 1.31

method	result

derivativedivides	$\frac{2a^4b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^3b^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{16}{3}a^4b+\frac{4}{3}a^2b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(6a^3b^2+2a\,b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{178}{15}a^4b+\cdots\right.}{}$
default	$\frac{2a^4b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^3b^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{16}{3}a^4b+\frac{4}{3}a^2b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(6a^3b^2+2a\,b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{178}{15}a^4b+\cdots\right.}{}$
risch	$-\frac{i\mathrm{e}^{3i(dx+c)}b}{32(-2iab+a^2-b^2)d}+\frac{5\,\mathrm{e}^{3i(dx+c)}a}{96(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{i(dx+c)}ab}{4(-3ib\,a^2+ib^3+a^3-3b^2a)d}-\frac{5\,\mathrm{e}^{i(dx+c)}a^2}{16(-3ib\,a^2+ib^3+a^3-3b^2a)d}+\frac{}{16\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(a^4*b*tan(1/2*d*x+1/2*c)^9+a^3*b^2*ta
n(1/2*d*x+1/2*c)^8+(16/3*a^4*b+4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(6*a^3*b^2
+2*a*b^4)*tan(1/2*d*x+1/2*c)^6+(178/15*a^4*b+136/15*a^2*b^3+16/5*b^5)*tan(1
/2*d*x+1/2*c)^5+(-16/3*a^5-2/3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(16/3*a^4*b+4/3*
a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(2*a^3*b^2-8/3*a^5+2/3*a*b^4)*tan(1/2*d*x+1/2
*c)^2+a^4*b*tan(1/2*d*x+1/2*c)-8/15*a^5+3/5*a^3*b^2+2/15*a*b^4)/(1+tan(1/2*
d*x+1/2*c)^2)^5-64*a^5*b/(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(260) = 520$.
time = 0.57, size = 658, normalized size = 2.40

$$\frac{\frac{15\,a^5b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(8\,a^5-9\,a^3b^2-2\,ab^4-\frac{15\,a^4b\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\,a^3b^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\,a^4b\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{10\left(4\,a^5-3\,a^3b^2-ab^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{20\left(4\,a^4b+a^2b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\left(8\,a^5+ab^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\left(89\,a^4b+68\,a^2b^3+24\,b^5\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{30\left(3\,a^3b^2+ab^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{20\left(4\,a^4b+a^2b^3\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6+\frac{5\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(15*a^5*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(8*a^5 - 9*a^3*b^2 - 2*a*b^4 -
15*a^4*b*sin(d*x + c)/(cos(d*x + c) + 1) - 15*a^3*b^2*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 - 15*a^4*b*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 10*(4*a^5 -
 3*a^3*b^2 - a*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 20*(4*a^4*b + a^2
*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(8*a^5 + a*b^4)*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 2*(89*a^4*b + 68*a^2*b^3 + 24*b^5)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 30*(3*a^3*b^2 + a*b^4)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*(4*a^4*b + a^2*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^6
```

```
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

Fricas [A]
time = 0.37, size = 370, normalized size = 1.35

$$\frac{15\sqrt{a^2+b^2}\,a^5 b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-6\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(dx+c)^5+10\,(2\,a^7+5\,a^5b^2+4\,a^3b^4+ab^6)\cos(dx+c)^3-30\,(a^7+a^5b^2)\cos(dx+c)+2\,\left(23\,a^6b+34\,a^4b^3+14\,a^2b^5+3\,b^7+3\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos(dx+c)^4-(11\,a^6b+28\,a^4b^3+23\,a^2b^5+6\,b^7)\cos(dx+c)^2\right)\sin(dx+c)}{30\,(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^5*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a
*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^5 + 10*(2
*a^7 + 5*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 30*(a^7 + a^5*b^2)*c
os(d*x + c) + 2*(23*a^6*b + 34*a^4*b^3 + 14*a^2*b^5 + 3*b^7 + 3*(a^6*b + 3*
a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - (11*a^6*b + 28*a^4*b^3 + 23*a^2
*b^5 + 6*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 464, normalized size = 1.69

$$\frac{\frac{15\,a^5b\log\left(\left|\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\sqrt{a^2+b^2}}{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\sqrt{a^2+b^2}}\right|\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+90\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+30\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+178\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+136\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+48\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^5+9\,a^3b^2+2\,ab^4\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9 +
```

```
15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 20*a^
2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*a*b^4
*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/2*d*x +
 1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x + 1/2*c)
^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*c)^2 + 30
*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*
b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```

Mupad [B]
time = 6.83, size = 683, normalized size = 2.49

$$\frac{\frac{2\,\left(-8\,a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{15\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a^4\,b+a^2\,b^3\right)}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(3\,a^3\,b^2+a\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-4\,a^5+3\,a^3\,b^2+a\,b^4\right)}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(89\,a^4\,b+68\,a^2\,b^3+24\,b^5\right)}{15\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^5+a\,b^4\right)}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,a^3\,b^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{8\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(4\,a^4+a^2\,b^2\right)}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,a^4\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,a^4\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a^5\,b\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}{2\,{\left(a^2+b^2\right)}^{7/2}}\right)}{d\,{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*tan(c + d*x)),x)
```

```
[Out] ((2*(2*a*b^4 - 8*a^5 + 9*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (8*tan(c/2 + (d*x)/2)^3*(4*a^4*b + a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 3*a^3*b^2))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) + (4*tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^5 + 3*a^3*b^2))
/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^5*(89*a^4*
b + 24*b^5 + 68*a^2*b^3))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*tan
(c/2 + (d*x)/2)^4*(a*b^4 + 8*a^5))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (2*a^3*b^2*tan(c/2 + (d*x)/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (8
*b*tan(c/2 + (d*x)/2)^7*(4*a^4 + a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (2*a^4*b*tan(c/2 + (d*x)/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
+ (2*a^4*b*tan(c/2 + (d*x)/2)^9)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(5
*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 +
 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (2*a^5*b*atanh((2*a
^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^2)^(7/2))
```

3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{a(3a^4-6a^2b^2-b^4)\,x}{8\,(a^2+b^2)^3}+\frac{a^4b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}-\frac{\cos^2(c+dx}{}$$

```
[Out] 1/8*a*(3*a^4-6*a^2*b^2-b^4)*x/(a^2+b^2)^3+a^4*b*ln(a*cos(d*x+c)+b*sin(d*x+c
))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d-1/8*cos(d*x+
c)^2*(4*b*(2*a^2+b^2)+a*(5*a^2+b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 815, 649, 209, 266}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)}-\frac{\cos^2(c+dx)\,(a(5a^2+b^2)\tan(c+dx)+4b(2a^2+b^2))}{8d\,(a^2+b^2)^2}+\frac{a^4b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{ax(3a^4-6a^2b^2-b^4)}{8\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*
x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^
2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2+b^2}-\frac{3ab^4x}{a^2+b^2}-4b^2x^2}{(a+x)(b^2+x^2)^2}\,dx, x, b\tan(c+dx\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(\right.}{8(a^2+b^2)^2d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(\right.}{8(a^2+b^2)^2d}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b\right.}{}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b\right.}{}$$

$$= \frac{a(3a^4-6a^2b^2-b^4)x}{8(a^2+b^2)^3} + \frac{a^4b\log(\cos(c+dx))}{(a^2+b^2)^3d} + \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3d} + \frac{\cos}{}$$

Mathematica [A]
time = 2.96, size = 249, normalized size = 1.58

$$-\frac{2ab(5a^4+6a^2b^2+b^4)\operatorname{ArcTan}(\tan(c+dx))+8b^2(2a^4+3a^2b^2+b^4)\cos^2(c+dx)-4b^2(a^2+b^2)^2\cos^4(c+dx)+8a^4\left(\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2b^2\log(a+b\tan(c+dx))+\left(b^2-a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)-4ab(a^2+b^2)^2\cos^3(c+dx)\sin(c+dx)+a(5a^4b+6a^2b^3+b^5)\sin(2(c+dx))}{16b\,(a^2+b^2)^3\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/16*(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4
+ 3*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 + 8*
a^4*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a +
b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]) -
4*a*b*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x] + a*(5*a^4*b + 6*a^2*b^3 +
b^5)*Sin[2*(c + d*x)])/(b*(a^2 + b^2)^3*d)
```

Maple [A]
time = 0.30, size = 208, normalized size = 1.32

method	result
derivativedivides	$\dfrac{\dfrac{b\,a^4\ln(a+b\tan(dx+c))}{(a^2+b^2)^3}+\dfrac{\dfrac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}a\,b^4\right)\tan(}{\left(1+\tan^2(dx+c)\right)^2}}{(a^2+b^2)^3}}{d}$
default	$\dfrac{\dfrac{b\,a^4\ln(a+b\tan(dx+c))}{(a^2+b^2)^3}+\dfrac{\dfrac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}a\,b^4\right)\tan(}{\left(1+\tan^2(dx+c)\right)^2}}{(a^2+b^2)^3}}{d}$
risch	$\frac{iabx}{24ib\,a^2-8ib^3-8a^3+24b^2a}-\frac{3a^2x}{8(3ib\,a^2-ib^3-a^3+3b^2a)}+\frac{e^{2i(dx+c)}b}{16(-2iab+a^2-b^2)d}+\frac{ie^{2i(dx+c)}a}{8(-2iab+a^2-b^2)d}+\frac{e^{-2i(dx+c)}b}{16(ib+a)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*a^4/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(((-5/8*a^5-3/4*a^3
*b^2-1/8*a*b^4)*tan(d*x+c)^3+(-a^4*b-3/2*a^2*b^3-1/2*b^5)*tan(d*x+c)^2+(-3/
8*a^5-1/4*a^3*b^2+1/8*a*b^4)*tan(d*x+c)-3/4*a^4*b-a^2*b^3-1/4*b^5)/(1+tan(d
*x+c)^2)^2+1/8*a*(-4*a^3*b*ln(1+tan(d*x+c)^2)+(3*a^4-6*a^2*b^2-b^4)*arctan(
tan(d*x+c)))))
```

Maxima [A]
time = 0.56, size = 280, normalized size = 1.77

$$\frac{\frac{8\,a^4b\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^5-6\,a^3b^2-ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(5\,a^3+ab^2)\tan(dx+c)^3+6\,a^2b+2\,b^3+4\,(2\,a^2b+b^3)\tan(dx+c)^2+(3\,a^3-ab^2)\tan(dx+c)}{(a^4+2\,a^2b^2+b^4)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\,(a^4+2\,a^2b^2+b^4)\tan(dx+c)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/8*(8*a^4*b*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*tan(d*x + c)^2 + (3*a^3 - a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d

Fricas [**A**]
time = 0.37, size = 216, normalized size = 1.37

$$\frac{4\,a^4b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\,\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\left(3\,a^5-6\,a^3b^2-ab^4\right)dx-4\,\left(2\,a^4b+3\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+\left(2\,\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^3-\left(5\,a^5+6\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b^2 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(152) = 304.
time = 0.45, size = 334, normalized size = 2.11

$$\frac{\frac{8\,a^4b^2\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,a^4b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,a^4b\tan\left(dx+c\right)^4-5\,a^5\tan\left(dx+c\right)^3-6\,a^3b^2\tan\left(dx+c\right)^3-ab^4\tan\left(dx+c\right)^3+4\,a^4b\tan\left(dx+c\right)^2-12\,a^2b^3\tan\left(dx+c\right)^2-4\,b^5\tan\left(dx+c\right)^2-3\,a^5\tan\left(dx+c\right)-2\,a^3b^2\tan\left(dx+c\right)+ab^4\tan\left(dx+c\right)-8\,a^2b^3-2\,b^5}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(\tan\left(dx+c\right)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x + c

```
)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d*x +
c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x + c) +
 a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*(tan(d*x + c)^2 + 1)^2))/d
```

Mupad [B]
time = 4.27, size = 313, normalized size = 1.98

$$\frac{a^4\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^3}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a\,b-a^2\,3\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,1\mathrm{i}\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\frac{3\,a^2\,b+b^3}{4\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\tan(c+d\,x)}^3\,\left(5\,a^3+a\,b^2\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\tan(c+d\,x)}^2\,\left(2\,a^2\,b+b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\tan(c+d\,x)\,\left(3\,a^2-b^2\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(c+d\,x\right)}^4+2\,{\tan\left(c+d\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^4*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)
*(a*b - a^2*3i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c +
d*x) + 1i)*(a*b*1i - 3*a^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - (
(3*a^2*b + b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(a*b^2 + 5*a^
3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^2*(2*a^2*b + b^3))/(2*(a^4
+ b^4 + 2*a^2*b^2)) + (a*tan(c + d*x)*(3*a^2 - b^2))/(8*(a^4 + b^4 + 2*a^2*
b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=168

$$\frac{a^3 b \tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2} d}+\frac{a b^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right) d}+\frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right) d}+\frac{a^2 b\sin(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\sin^3}{3\left(a^2\right.}$$

[Out] a^3*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x+c)/(a^2+b^2)/d+1/3*a*cos(d*x+c)^3/(a^2+b^2)/d+a^2*b*sin(d*x+c)/(a^2+b^2)^2/d+1/3*b*sin(d*x+c)^3/(a^2+b^2)/d

Rubi [A]
time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3599, 3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{a^2 b\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{a b^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^3 b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/(
(a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)\sin^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \frac{a\int \sin^3(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^2(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx}}{a^2+b^2}$$

$$= \frac{a^2b\sin(c+dx)}{(a^2+b^2)^2\,d} - \frac{(a^3b)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \sin(c+dx)\,dx}{(a^2+b^2)^2} - \frac{a\mathrm{Sub}}{}$$

$$= \frac{ab^2\cos(c+dx)}{(a^2+b^2)^2\,d} - \frac{a\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\cos^3(c+dx)}{3\,(a^2+b^2)\,d} + \frac{a^2b\sin(c+dx)}{(a^2+b^2)^2\,d} + \frac{b\sin^3(c+}{3\,(a^2+b^2}$$

$$= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{ab^2\cos(c+dx)}{(a^2+b^2)^2\,d} - \frac{a\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\cos^3(c}{3\,(a^2+}$$

Mathematica [A]

time = 1.58, size = 139, normalized size = 0.83

$$\frac{-24a^3b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}\,((-9a^3+3ab^2)\cos(c+dx) + a(a^2+b^2)\cos(3(c+dx)) - 2b(-7a^2-b^2+(a^2+b^2)\cos(2(c+dx)))\sin(c+dx))}{12\,(a^2+b^2)^{5/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] - 2*
b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b
^2)^(5/2)*d)
```

Maple [A]

time = 0.32, size = 202, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4a^3}{3}+\frac{2b^2}{3}}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$
default	$\frac{\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4a^3}{3}+\frac{2b^2}{3}}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$

risch	$\frac{ie^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3\,e^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{ie^{-i(dx+c)}b}{8(ib+a)^2d}-\frac{3\,e^{-i(dx+c)}a}{8(ib+a)^2d}-\frac{ib\,a^3\ln\left(e^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^4+2*a^2*b^2+b^4)*(a^2*b*tan(1/2*d*x+1/2*c)^5+b^2*a*tan(1/2*d*x+1/
2*c)^4+(10/3*a^2*b+4/3*b^3)*tan(1/2*d*x+1/2*c)^3-2*a^3*tan(1/2*d*x+1/2*c)^2
+a^2*b*tan(1/2*d*x+1/2*c)-2/3*a^3+1/3*b^2*a)/(1+tan(1/2*d*x+1/2*c)^2)^3-16*
a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(160) = 320$.
time = 0.57, size = 364, normalized size = 2.17

$$\frac{\frac{3\,a^3b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3-ab^2-\frac{3\,a^2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\,ab^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,a^2b\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\left(5\,a^2b+2\,b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^
2 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3 - a*b^2 - 3*a^2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 3*a^2*b*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2*(5*a^2*b + 2*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^2*b^2 +
 b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*(a
^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^
2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [A]
time = 0.35, size = 261, normalized size = 1.55

$$\frac{3\,\sqrt{a^2+b^2}\,a^3b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^5+2\,a^3b^2+ab^4)\cos(dx+c)^3-6\,(a^5+a^3b^2)\cos(dx+c)+2\left(4\,a^4b+5\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^2\right)\sin(dx+c)}{6\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*s
```

```
in(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^2)*c
os(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A]

time = 0.47, size = 241, normalized size = 1.43

$$\frac{\frac{3\,a^3b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^3+ab^2\right)}{(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(1/2
*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2
*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3
 + a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B]

time = 6.50, size = 324, normalized size = 1.93

$$\frac{\frac{\frac{2\,a\,b^2}{3}-\frac{4\,a^3}{3}}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{20\,a^2\,b}{3}+\frac{8\,b^3}{3}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] (((2*a*b^2)/3 - (4*a^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((20*a^2*b)/3 + (8*b
^3)/3))/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 +
 2*a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a
^2*b*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 + 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2
)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^3*b*atanh(
(a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a
^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{a(a^2-b^2)\,x}{2\,(a^2+b^2)^2}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d}$$

```
[Out] 1/2*a*(a^2-b^2)*x/(a^2+b^2)^2+a^2*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)
^2/d-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Rubi [A]

time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 815, 649, 209, 266}

$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{ax(a^2-b^2)}{2\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{(a+x)(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)} - \frac{ab^2(a^2-b^2-2ax)}{(a^2+b^2)^2(b^2+x^2)}\right) \right.}{2bd}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} + \frac{(ab)\text{Subst}\left(\int \frac{a^2-}{b}\right.}{2}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{}{b^2}\right.}{(a}$$

$$= \frac{a(a^2-b^2)\,x}{2\left(a^2+b^2\right)^2} + \frac{a^2b\log(\cos(c+dx))}{\left(a^2+b^2\right)^2d} + \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{\cos^2(c+dx)}{2(}$$

Mathematica [A]

time = 0.82, size = 170, normalized size = 1.81

$$-\frac{2ab(a^2+b^2)\,\text{ArcTan}(\tan(c+dx)) + 2b^2(a^2+b^2)\cos^2(c+dx) + a\left(2a\left(\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) - 2b^2\log(a+b\tan(c+dx)) + \left(b^2-a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right) + b(a^2+b^2)\sin(2(c+dx))\right)}{4b\left(a^2+b^2\right)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/4*(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d*
x]^2 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^
2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)]))/(b*(a^2 + b^2)^2*d)
```

Maple [A]
time = 0.26, size = 122, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{a^{2} b \ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(-\frac{1}{2}a^{3}-\frac{1}{2}b^{2}a\right)\tan\left(dx+c\right)-\frac{a^{2}b}{2}-\frac{b^{3}}{2}}{1+\tan^{2}\left(dx+c\right)}+\frac{a\left(-ab\ln\left(1+\tan^{2}\left(dx+c\right)\right)+\left(a^{2}-b^{2}\right)\arctan\left(\tan\left(dx+c\right)\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
default	$\frac{\frac{a^{2} b \ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(-\frac{1}{2}a^{3}-\frac{1}{2}b^{2}a\right)\tan\left(dx+c\right)-\frac{a^{2}b}{2}-\frac{b^{3}}{2}}{1+\tan^{2}\left(dx+c\right)}+\frac{a\left(-ab\ln\left(1+\tan^{2}\left(dx+c\right)\right)+\left(a^{2}-b^{2}\right)\arctan\left(\tan\left(dx+c\right)\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
risch	$-\frac{ax}{2(2iab-a^{2}+b^{2})}+\frac{i e^{2i(dx+c)}}{8(-ib+a)d}-\frac{i e^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ia^{2}bx}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{2ia^{2}bc}{d(a^{4}+2a^{2}b^{2}+b^{4})}+\frac{a^{2}b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^{4}+2a^{2}b^{2}+b^{4})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(((-1/2*a^3-1/2*b^2
*a)*tan(d*x+c)-1/2*a^2*b-1/2*b^3)/(1+tan(d*x+c)^2)+1/2*a*(-a*b*ln(1+tan(d*x
+c)^2)+(a^2-b^2)*arctan(tan(d*x+c)))))
```

Maxima [A]
time = 0.62, size = 144, normalized size = 1.53

$$\frac{\frac{2\,a^{2}b\log\left(b\tan\left(dx+c\right)+a\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{a^{2}b\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{3}-ab^{2}\right)\left(dx+c\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{a\tan\left(dx+c\right)+b}{\left(a^{2}+b^{2}\right)\tan\left(dx+c\right)^{2}+a^{2}+b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(ta
n(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) - (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 +
 b^2))/d
```

Fricas [A]
time = 0.39, size = 122, normalized size = 1.30

$$\frac{a^{2}b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)+\left(a^{3}-ab^{2}\right)dx-\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}-\left(a^{3}+ab^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*cos(d*x + c)^2 - (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.
time = 0.45, size = 184, normalized size = 1.96

$$\frac{\frac{2\,a^2b^2\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^3-ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{a^2b\tan(dx+c)^2-a^3\tan(dx+c)-ab^2\tan(dx+c)-b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^2
*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a*b^
2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

Mupad [B]
time = 3.87, size = 147, normalized size = 1.56

$$\frac{a^2\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^2}-\frac{a\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2\,(a^2+b^2)}+\frac{a\tan(c+d\,x)}{2\,(a^2+b^2)}\right)}{d}-\frac{a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^2*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2) - (a*log(tan(c + d*x) + 1
i)*1i)/(4*d*(a*b*2i - a^2 + b^2)) - (a*log(tan(c + d*x) - 1i))/(4*d*(2*a*b
- a^2*1i + b^2*1i)) - (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))
/(2*(a^2 + b^2))))/d
```

3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}$$

```
[Out] a*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
a*cos(d*x+c)/(a^2+b^2)/d+b*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3599, 3188, 2717, 2718, 3153, 212}

$$\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b
^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx\\ &= \frac{a\int \sin(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}\\ &= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}+\frac{(ab)\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\text{s}\right.}{\left(a^2+b^2\right)d}\\ &= \frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 79, normalized size = 0.88

$$\frac{-2ab\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\,(-a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A]
time = 0.27, size = 101, normalized size = 1.12

method	result	size
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
risch	$-\frac{e^{i(dx+c)}}{2(-ib+a)d}-\frac{e^{-i(dx+c)}}{2(ib+a)d}+\frac{iba\ln\left(e^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}-\frac{iba\ln\left(e^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}$	169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)*(b*tan(1/2*d*x+1/2*c)-a)/(1+tan(1/2*d*x+1/2*c)^2)-4*a*b/(2
*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b
^2)^(1/2)))
```

Maxima [A]
time = 0.63, size = 141, normalized size = 1.57

$$\frac{\frac{ab\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a
 - b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(86) = 172$.
time = 0.39, size = 185, normalized size = 2.06

$$\frac{\sqrt{a^2+b^2}\,ab\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2\left(a^3+ab^2\right)\cos(dx+c)+2\left(a^2b+b^3\right)\sin(dx+c)}{2\left(a^4+2a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)) - 2*(a^3 + a*b^2)*cos(d*x + c) + 2*(a^2*b + b^3)*sin(d*x + c))/((a^4 +
 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 118, normalized size = 1.31

$$\frac{\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan
(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B]
time = 3.91, size = 110, normalized size = 1.22

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] (2*a*b*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(a^2 + b^2)^(
3/2)))/(d*(a^2 + b^2)^(3/2)) - ((2*a)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)
)/(a^2 + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

[Out] -arctanh(cos(d*x+c))/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/a/d/(a^2+b^2)^(1/2)

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3599, 3189, 3855, 3153, 212}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]

[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \int \left(\frac{\csc(c+dx)}{a} - \frac{b}{a(a\cos(c+dx)+b\sin(c+dx))}\right)dx \\
&= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 75, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
- Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

Maple [A]
time = 0.31, size = 63, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b
^2)^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.64, size = 107, normalized size = 1.62

$$\frac{\frac{b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin
(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(
sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(62) = 124.
time = 0.41, size = 183, normalized size = 2.77

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-(a^2+b^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a^2+b^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x
```

```
 + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos(d*x
 + c) + 1/2))/((a^3 + a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)
[Out] Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 94, normalized size = 1.42

$$\frac{\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] (b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(
tan(1/2*d*x + 1/2*c)))/a)/d
```

Mupad [B]
time = 4.09, size = 174, normalized size = 2.64

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)}{b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4\mathrm{i}+a\,b^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+a^2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*tan(c + d*x))),x)
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - (2*b*atanh(((a^2 + b^2)^
(1/2)*(a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x)/2)*4i + a*b*cos(c/2
+ (d*x)/2)*2i))/(b^3*sin(c/2 + (d*x)/2)*4i + a*b^2*cos(c/2 + (d*x)/2)*1i +
a^2*b*sin(c/2 + (d*x)/2)*3i + a*cos(c/2 + (d*x)/2)*(a^2 + b^2)*1i)))/(a*d*(
a^2 + b^2)^(1/2))
```

3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}$$

```
[Out] -cot(d*x+c)/a/d-b*ln(tan(d*x+c))/a^2/d+b*ln(a+b*tan(d*x+c))/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 46}

$$-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) - (b*Log[Tan[c + d*x]])/(a^2*d) + (b*Log[a + b*Tan[c
+ d*x]])/(a^2*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)\,dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 47, normalized size = 0.94

$$\frac{-a\cot(c+dx)+b(-\log(\sin(c+dx))+\log(a\cos(c+dx)+b\sin(c+dx)))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-(a*Cot[c + d*x]) + b*(-Log[Sin[c + d*x]] + Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]))/(a^2*d)
```

Maple [A]
time = 0.25, size = 48, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{b\ln(a+b\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}}{d}$	48
default	$\frac{\frac{b\ln(a+b\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}}{d}$	48
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^2d}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/a^2*ln(a+b*tan(d*x+c))-1/a/tan(d*x+c)-b/a^2*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.34, size = 47, normalized size = 0.94

$$\frac{\frac{b\log(b\tan(dx+c)+a)}{a^2}-\frac{b\log(\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*tan(d*x + c) + a)/a^2 - b*log(tan(d*x + c))/a^2 - 1/(a*tan(d*x + c
)))/d
```

Fricas [A]
time = 0.36, size = 95, normalized size = 1.90

$$\frac{b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)}{2\,a^2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 2*a*cos(
d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 60, normalized size = 1.20

$$\frac{\frac{b\log(|b\tan(dx+c)+a|)}{a^2} - \frac{b\log(|\tan(dx+c)|)}{a^2} + \frac{b\tan(dx+c)-a}{a^2\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*tan
(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

Mupad [B]
time = 3.74, size = 39, normalized size = 0.78

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^2\,d} - \frac{\cot(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^2*d) - cot(c + d*x)/(a*d)
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)} \, dx$

Optimal. Leaf size=122

$$-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*csc(d*x+c)/a^2
/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2
+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3/d
```

Rubi [A]
time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3599, 3189, 3853, 3855, 2701, 327, 213, 2702, 3183, 3153, 212}

$$-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b
*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]
)/(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b\csc^2(c+dx)\sec(c+dx)}{a^2} + \frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{}{a}\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c+dx)\sec^2(c+d}{a^3}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{b^2\sec(c+dx)}{a^3d} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{b\int \sec(c+dx)}{a^2}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{b\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\,\mathrm{cs}}{2ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx}{\sqrt{a}}\right.}{a^3d}$$

Mathematica [A]
time = 0.86, size = 179, normalized size = 1.47

$$\frac{-16b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+4ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)-4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+4ab\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x)/
2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log
[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(8*a^
3*d)
```

Maple [A]
time = 0.38, size = 140, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{4a^3}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{4a^3}}{d}$
risch	$\frac{i\left(-ia\,e^{3i(dx+c)}+2b\,e^{3i(dx+c)}-ia\,e^{i(dx+c)}-2\,e^{i(dx+c)}b\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}-1\right)b^2}{a^3d}-\frac{i\sqrt{-a^2-b^2}\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c))-2*b*(a^2+b
^2)^(1/2)/a^3*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/8
/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2*b/
a^2/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.56, size = 215, normalized size = 1.76

$$\frac{\frac{\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{4\left(a^2+2\,b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}+\frac{8\left(a^2b+b^3\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*((4*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/a^2 + 4*(a^2 + 2*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (a
 - 4*b*sin(d*x + c)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)^2/(a^2*sin(d*x +
 c)^2) + 8*(a^2*b + b^3)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*a^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(114) = 228.
time = 0.39, size = 270, normalized size = 2.21

$$\frac{2\,a^2\cos(dx+c)-4\,ab\sin(dx+c)+2\left(b\cos(dx+c)^2-b\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-\left(\left(a^2+2\,b^2\right)\cos(dx+c)^2-a^2-2\,b^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\left(a^2+2\,b^2\right)\cos(dx+c)^2-a^2-2\,b^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c) - 4*a*b*sin(d*x + c) + 2*(b*cos(d*x + c)^2 - b)*sqr
t(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2
*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*
d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 209, normalized size = 1.71

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{4\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8\,(a^2b+b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}-\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8*(a^2*b + b^3)*log(abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (6*a^2*tan(1/2*d*x + 1/2*
c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a
^3*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 4.45, size = 764, normalized size = 6.26

$$\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2\left(\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}\right)}-\frac{a^2\left(\frac{\cos(c+dx)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{4}\right)}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}+\frac{a\,b\sin(c+dx)}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}-\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{2\left(\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}\right)}+\frac{b\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,1i+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8i+a\,b^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,4i+a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,3i+a^2b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2+12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a\,b^4+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^5}\right)\sqrt{a^2+b^2}\,1i}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}-\frac{b\cos(2c+2dx)\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,1i+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8i+a\,b^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,4i+a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,3i+a^2b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2+12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a\,b^4+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^5}\right)\sqrt{a^2+b^2}\,1i}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

```
[Out] (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 - log(sin(c/2 + (d*x)/2)/cos(c/2
```

```
+ (d*x)/2))/4 + (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) + (a*b*sin(c + d*x))/((a^3*
d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2)) +
(b*atan((a^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/
2)*(a^2 + b^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a
^3*b*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(
a^2 + b^2)^(1/2)*8i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4
*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2
+ (d*x)/2) + 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)
/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a^4*sin(c/2 +
(d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i
 + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a^3*b*cos(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i)/(a
^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/
2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2 + (d*x)/2) + 12*a^2*b^3
*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)/2 - (a^3*d*cos(2*c + 2
*d*x))/2)
```

3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(a^2+b^2)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/a^3/d+1/2*b*cot(d*x+c)^2/a^2/d-1/3*cot(d*x+c)^3/a/d-b
*(a^2+b^2)*ln(tan(d*x+c))/a^4/d+b*(a^2+b^2)*ln(a+b*tan(d*x+c))/a^4/d
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{b\cot^2(c+dx)}{2a^2d}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}-\frac{(a^2+b^2)\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[
c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 +
b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{ax^4} - \frac{b^2}{a^2x^3} + \frac{a^2+b^2}{a^3x^2} + \frac{-a^2-b^2}{a^4x} + \frac{a^2+b^2}{a^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2+b^2)\cot(c+dx)}{a^3d} + \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} - \frac{b(a^2+b^2)\log(\tan(c+}{a^4d}$$

Mathematica [A]
time = 0.53, size = 95, normalized size = 0.88

$$\frac{3a^2b\csc^2(c+dx) - 2\cot(c+dx)\left(2a^3+3ab^2+a^3\csc^2(c+dx)\right) - 6b(a^2+b^2)\left(\log(\sin(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx))\right)}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a^2*b*Csc[c + d*x]^2 - 2*Cot[c + d*x]*(2*a^3 + 3*a*b^2 + a^3*Csc[c + d*x
]^2) - 6*b*(a^2 + b^2)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
d*x]]))/(6*a^4*d)
```

Maple [A]
time = 0.28, size = 96, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\left(a^2+b^2\right)b\ln(a+b\tan(dx+c))}{a^4} - \frac{1}{3a\tan(dx+c)^3} - \frac{a^2+b^2}{a^3\tan(dx+c)} + \frac{b}{2a^2\tan(dx+c)^2} - \frac{\left(a^2+b^2\right)b\ln(\tan(dx+c))}{a^4}}{d}$
default	$\frac{\frac{\left(a^2+b^2\right)b\ln(a+b\tan(dx+c))}{a^4} - \frac{1}{3a\tan(dx+c)^3} - \frac{a^2+b^2}{a^3\tan(dx+c)} + \frac{b}{2a^2\tan(dx+c)^2} - \frac{\left(a^2+b^2\right)b\ln(\tan(dx+c))}{a^4}}{d}$
risch	$-\frac{2\left(3ib^2e^{4i(dx+c)}+3ab\,e^{4i(dx+c)}-6ia^2e^{2i(dx+c)}-6ib^2e^{2i(dx+c)}-3ab\,e^{2i(dx+c)}+2ia^2+3ib^2\right)}{3d\,a^3\left(e^{2i(dx+c)}-1\right)^3} - \frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d} - b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a^2+b^2)/a^4*b*ln(a+b*tan(d*x+c))-1/3/a/tan(d*x+c)^3-(a^2+b^2)/a^3/ta
n(d*x+c)+1/2*b/a^2/tan(d*x+c)^2-(a^2+b^2)/a^4*b*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.33, size = 97, normalized size = 0.90

$$\frac{\frac{6\left(a^2b+b^3\right)\log(b\tan(dx+c)+a)}{a^4} - \frac{6\left(a^2b+b^3\right)\log(\tan(dx+c))}{a^4} + \frac{3\,ab\tan(dx+c)-6\left(a^2+b^2\right)\tan(dx+c)^2-2\,a^2}{a^3\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^2*b + b^3)*log(b*tan(d*x + c) + a)/a^4 - 6*(a^2*b + b^3)*log(tan(
d*x + c))/a^4 + (3*a*b*tan(d*x + c) - 6*(a^2 + b^2)*tan(d*x + c)^2 - 2*a^2)
/(a^3*tan(d*x + c)^3))/d
```

Fricas [A]
time = 0.41, size = 208, normalized size = 1.93

$$-\frac{2\,(2\,a^3+3\,ab^2)\cos(dx+c)^3+3\,a^2b\sin(dx+c)+3\,(a^2b+b^3-(a^2b+b^3)\cos(dx+c)^2)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)-3\,(a^2b+b^3-(a^2b+b^3)\cos(dx+c)^2)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)-6\,(a^3+ab^2)\cos(dx+c)}{6\,(a^4d\cos(dx+c)^2-a^4d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 3*a^2*b*sin(d*x + c) + 3*(a^2*b
+ b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c) - 3*(a^2*b + b^3 - (a^2*b +
 b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*(a^3
+ a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 144, normalized size = 1.33

$$-\frac{\frac{6\,(a^2b+b^3)\log(|\tan(dx+c)|)}{a^4}-\frac{6\,(a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{a^4b}-\frac{11\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3-6\,a^3\tan(dx+c)^2-6\,ab^2\tan(dx+c)^2+3\,a^2b\tan(dx+c)-2\,a^3}{a^4\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(a^2*b + b^3)*log(abs(tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*log(ab
s(b*tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x
+ c)^3 - 6*a^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x +
c) - 2*a^3)/(a^4*tan(d*x + c)^3))/d
```

Mupad [B]
time = 3.84, size = 102, normalized size = 0.94

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\left(a^2+b^2\right)\,\left(a+2\,b\,\tan\left(c+d\,x\right)\right)}{a\,\left(a^2\,b+b^3\right)}\right)\,\left(a^2+b^2\right)}{a^4\,d}-\frac{\frac{1}{3\,a}+\frac{{\tan\left(c+d\,x\right)}^2\,\left(a^2+b^2\right)}{a^3}-\frac{b\,\tan\left(c+d\,x\right)}{2\,a^2}}{d\,{\tan\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(a^2*b + b^3)))*(a^2 +
 b^2))/(a^4*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + b^2))/a^3 - (b*tan(c + d
*x))/(2*a^2))/(d*tan(c + d*x)^3)
```

3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=169

$$-\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d}+\frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d}+\frac{b\cot^4(c+dx)}{4a^2d}-\frac{\cot^5(c+dx)}{5ad}-$$

```
[Out] -(a^2+b^2)^2*cot(d*x+c)/a^5/d+1/2*b*(2*a^2+b^2)*cot(d*x+c)^2/a^4/d-1/3*(2*a
^2+b^2)*cot(d*x+c)^3/a^3/d+1/4*b*cot(d*x+c)^4/a^2/d-1/5*cot(d*x+c)^5/a/d-b*
(a^2+b^2)^2*ln(tan(d*x+c))/a^6/d+b*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/a^6/d
```

Rubi [A]
time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{b\cot^4(c+dx)}{4a^2d}-\frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d}+\frac{b(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6d}-\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d}+\frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/
(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(
4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^
6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{2a^2b^2+b^4}{a^3x^4} + \frac{b^2(-2a^2-b^2)}{a^4x^3} + \frac{(a^2+b^2)^2}{a^5x^2} - \frac{(a^2+b^2)^2}{a^6x} + \frac{(a^2+b^2)^2}{a^6(a+x)}\right) dx}{d}$$

$$= -\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d} + \frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d} - \frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d} + \frac{b}{}$$

Mathematica [A]
time = 2.20, size = 150, normalized size = 0.89

$$\frac{-4\cot(c+dx)\left(8a^5+25a^3b^2+15ab^4+a^3(4a^2+5b^2)\csc^2(c+dx)+3a^5\csc^4(c+dx)\right)+15b\left(2a^2(a^2+b^2)\csc^2(c+dx)+a^4\csc^4(c+dx)-4(a^2+b^2)^2(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))\right)}{60a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[c
 + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^2
+ a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c + d
*x] + b*Sin[c + d*x]])))/(60*a^6*d)
```

Maple [A]
time = 0.30, size = 165, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\left(a^4+2a^2b^2+b^4\right)b\ln(a+b\tan(dx+c))}{a^6} - \frac{1}{5a\tan(dx+c)^5} - \frac{2a^2+b^2}{3a^3\tan(dx+c)^3} - \frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)} + \frac{b}{4a^2\tan(dx+c)^4} + \frac{\left(2a^2+b^2\right)b}{2a^4\tan(dx+c)^2} - \frac{(a^4+}{}}{d}$
default	$\frac{\frac{\left(a^4+2a^2b^2+b^4\right)b\ln(a+b\tan(dx+c))}{a^6} - \frac{1}{5a\tan(dx+c)^5} - \frac{2a^2+b^2}{3a^3\tan(dx+c)^3} - \frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)} + \frac{b}{4a^2\tan(dx+c)^4} + \frac{\left(2a^2+b^2\right)b}{2a^4\tan(dx+c)^2} - \frac{(a^4+}{}}{d}$
risch	$-\frac{2i(-15ia^3b\,e^{8i(dx+c)}+15ia^3b\,e^{2i(dx+c)}+15a^2b^2e^{8i(dx+c)}+15b^4e^{8i(dx+c)}+45ia\,b^3e^{6i(dx+c)}+15ia\,b^3e^{2i(dx+c)}-90a^2b^2e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a^4+2*a^2*b^2+b^4)/a^6*b*ln(a+b*tan(d*x+c))-1/5/a/tan(d*x+c)^5-1/3*(2
*a^2+b^2)/a^3/tan(d*x+c)^3-(a^4+2*a^2*b^2+b^4)/a^5/tan(d*x+c)+1/4*b/a^2/tan
(d*x+c)^4+1/2*(2*a^2+b^2)/a^4*b/tan(d*x+c)^2-(a^4+2*a^2*b^2+b^4)/a^6*b*ln(t
an(d*x+c)))
```

Maxima [A]
time = 0.30, size = 168, normalized size = 0.99

$$\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^6}-\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log\left(\tan\left(dx+c\right)\right)}{a^6}+\frac{15\,a^3b\tan\left(dx+c\right)-60\left(a^4+2\,a^2b^2+b^4\right)\tan\left(dx+c\right)^4-12\,a^4+30\left(2\,a^3b+ab^3\right)\tan\left(dx+c\right)^3-20\left(2\,a^4+a^2b^2\right)\tan\left(dx+c\right)^2}{a^5\tan\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b
+ 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a^4
 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x
+ c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(161) = 322$.
time = 0.38, size = 385, normalized size = 2.28

$$-\frac{4\left(8\,a^5+25\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5-20\left(4\,a^5+11\,a^3b^2+6\,ab^4\right)\cos\left(dx+c\right)^3-30\left(a^4b+2\,a^2b^3+b^5+\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)+30\left(a^4b+2\,a^2b^3+b^5+\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+60\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)-15\left(3\,a^4b+2\,a^2b^3-2\left(a^4b+a^2b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\left(a^6d\cos\left(dx+c\right)^4-2\,a^6d\cos\left(dx+c\right)^2+a^6d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 20*(4*a^5 + 11*a^
3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*
a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)
*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*si
n(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cos(d*
x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c
)^2 + 1/4)*sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 15*(3
*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^
6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 251, normalized size = 1.49

$$-\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^6}-\frac{60\left(a^4b^2+2\,a^2b^4+b^6\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b}-\frac{137\,a^4b\tan\left(dx+c\right)^5+274\,a^2b^3\tan\left(dx+c\right)^5+137\,b^5\tan\left(dx+c\right)^5-60\,a^5\tan\left(dx+c\right)^4-120\,a^3b^2\tan\left(dx+c\right)^4-60\,ab^4\tan\left(dx+c\right)^4+60\,a^4b\tan\left(dx+c\right)^3+30\,a^2b^3\tan\left(dx+c\right)^3-40\,a^5\tan\left(dx+c\right)^2-20\,a^3b^2\tan\left(dx+c\right)^2+15\,a^4b\tan\left(dx+c\right)-12\,a^5}{a^6\tan\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(abs(tan(d*x + c)))/a^6 - 60*(a^4*b^
2 + 2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*tan(
d*x + c)^5 + 274*a^2*b^3*tan(d*x + c)^5 + 137*b^5*tan(d*x + c)^5 - 60*a^5*t
an(d*x + c)^4 - 120*a^3*b^2*tan(d*x + c)^4 - 60*a*b^4*tan(d*x + c)^4 + 60*a
^4*b*tan(d*x + c)^3 + 30*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^2 - 2
0*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 12*a^5)/(a^6*tan(d*x + c
)^5))/d
```

Mupad [B]
time = 4.33, size = 167, normalized size = 0.99

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,{\left(a^2+b^2\right)}^2\,\left(a+2\,b\,\tan(c+d\,x)\right)}{a\,\left(a^4\,b+2\,a^2\,b^3+b^5\right)}\right)\,{\left(a^2+b^2\right)}^2}{a^6\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^2\,\left(2\,a^2+b^2\right)}{3\,a^3}+\frac{\tan(c+d\,x)^4\,\left(a^4+2\,a^2\,b^2+b^4\right)}{a^5}-\frac{b\,\tan(c+d\,x)}{4\,a^2}-\frac{b\,\tan(c+d\,x)^3\,\left(2\,a^2+b^2\right)}{2\,a^4}}{d\,\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)^2*(a + 2*b*tan(c + d*x)))/(a*(a^4*b + b^5 + 2*a^2
*b^3)))*(a^2 + b^2)^2)/(a^6*d) - (1/(5*a) + (tan(c + d*x)^2*(2*a^2 + b^2))/
(3*a^3) + (tan(c + d*x)^4*(a^4 + b^4 + 2*a^2*b^2))/a^5 - (b*tan(c + d*x))/(
4*a^2) - (b*tan(c + d*x)^3*(2*a^2 + b^2))/(2*a^4))/(d*tan(c + d*x)^5)
```

3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=297

$$\frac{(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)\,x}{16\,(a^2+b^2)^5}+\frac{2a^5b(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}-\frac{a^6b}{(a^2+b^2)^4\,d(a+\dots}$$

```
[Out] 1/16*(5*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*x/(a^2+b^2)^5+2*a^5*b*(a^2
-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-a^6*b/(a^2+b^2)^4/d/(a+
b*tan(d*x+c))-1/6*cos(d*x+c)^6*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d+1
/24*cos(d*x+c)^4*(12*a*b*(3*a^2+b^2)+(13*a^4-18*a^2*b^2-7*b^4)*tan(d*x+c))/
(a^2+b^2)^3/d-1/16*cos(d*x+c)^2*(48*a^5*b+(11*a^6-43*a^4*b^2-7*a^2*b^4-b^6)
*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A]

time = 0.80, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{\cos^6(c+dx)\left((a^2-b^2)\tan(c+dx)+2ab\right)}{6d\,(a^2+b^2)^2}-\frac{a^6b}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}+\frac{2a^5b(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5}+\frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24d\,(a^2+b^2)^3}+\frac{x(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)}{16\,(a^2+b^2)^5}-\frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16d\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*x)/(16*(a^2 + b^2)^5)
+ (2*a^5*b*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)
^5*d) - (a^6*b)/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(2
*a*b + (a^2 - b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^2*d) + (Cos[c + d*x]^4*(12
*a*b*(3*a^2 + b^2) + (13*a^4 - 18*a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(24*(a^2
+ b^2)^3*d) - (Cos[c + d*x]^2*(48*a^5*b + (11*a^6 - 43*a^4*b^2 - 7*a^2*b^4
- b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^6}{(a+x)^2(b^2+x^2)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6(a^2-b^2)}{(a^2+b^2)^2}+\frac{2ab^6(5a^2+\cdots}{(a^2+b^2)\cdots}}{(a+\cdots}\right.}{\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+\cdots\right.}{24\,(\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+\cdots\right.}{24\,(\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+\cdots\right.}{24\,(\cdots}$$

$$= \frac{2a^5b(a^2-3b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos\cdots}{\cdots}$$

$$= \frac{2a^5b(a^2-3b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos\cdots}{\cdots}$$

$$= \frac{(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)\,x}{16\left(a^2+b^2\right)^5} + \frac{2a^5b(a^2-3b^2)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{2\cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 603 vs. 2(297) = 594.

time = 6.58, size = 603, normalized size = 2.03

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*(-1/2*((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 - b^6)*ArcTan[Tan[c + d*x]])/(b*(a
^2 + b^2)^4) - (3*a^5*Cos[c + d*x]^2)/(a^2 + b^2)^4 + (a*(3*a^2 + b^2)*Cos[
c + d*x]^4)/(2*(a^2 + b^2)^3) - (a*Cos[c + d*x]^6)/(3*(a^2 + b^2)^2) - (a^5
```

```
*(2*a^2 - 6*b^2 - (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*
x]])/(2*(a^2 + b^2)^5) + (2*a^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^2
 + b^2)^5 - (a^5*(2*a^2 - 6*b^2 + (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2
] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - ((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 -
b^6)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^4) + ((3*a^4 - 3*a^2*b^2 -
 2*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) - ((a^2 - b^2)*Cos
[c + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^2) + (3*(3*a^4 - 3*a^2*b^2 - 2*b
^4)*(ArcTan[Tan[c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^2
)^3) - (5*(a^2 - b^2)*((2*Cos[c + d*x]^3*Sin[c + d*x])/b + 3*b^2*(ArcTan[Ta
n[c + d*x]]/b^3 + (Cos[c + d*x]*Sin[c + d*x])/b^3)))/(48*(a^2 + b^2)^2) - a
^6/((a^2 + b^2)^4*(a + b*Tan[c + d*x]))))/d
```

Maple [A]
time = 0.49, size = 383, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))+2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5*ln
(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((-11/16*a^8+2*a^6*b^2+25/8*a^4*b^4+1/2*a^2
*b^6+1/16*b^8)*tan(d*x+c)^5+(-3*a^7*b-3*a^5*b^3)*tan(d*x+c)^4+(-5/6*a^8+13/
3*a^6*b^2+5*a^4*b^4-1/3*a^2*b^6-1/6*b^8)*tan(d*x+c)^3+(-9/2*a^7*b-5/2*a^5*b
^3+5/2*a^3*b^5+1/2*a*b^7)*tan(d*x+c)^2+(-5/16*a^8+2*a^6*b^2+15/8*a^4*b^4-1/
2*a^2*b^6-1/16*b^8)*tan(d*x+c)-11/6*a^7*b-1/2*a^5*b^3+3/2*a^3*b^5+1/6*a*b^7
)/(1+tan(d*x+c)^2)^3+1/32*(-32*a^7*b+96*a^5*b^3)*ln(1+tan(d*x+c)^2)+1/16*(5
*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(289) = 578.
time = 0.56, size = 799, normalized size = 2.69

$$\frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{96\,(a^7b-3\,a^5b^3)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{136\,a^6b-64\,a^4b^3-8\,a^2b^5+3\,(27\,a^6b-43\,a^4b^3-7\,a^2b^5-b^7)\tan(dx+c)^6+3\,(11\,a^7+5\,a^5b^2-7\,a^3b^4-ab^6)\tan(dx+c)^5+8\,(41\,a^6b-31\,a^4b^3+a^2b^5+b^7)\tan(dx+c)^4+8\,(5\,a^7-4\,a^5b^2-11\,a^3b^4-2\,ab^6)\tan(dx+c)^3+3\,(125\,a^6b-69\,a^4b^3-a^2b^5+b^7)\tan(dx+c)^2+(15\,a^7-23\,a^5b^2-43\,a^3b^4-5\,ab^6)\tan(dx+c)}{a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8+(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\tan(dx+c)^7+(a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)\tan(dx+c)^6+3\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\tan(dx+c)^5+3\,(a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)\tan(dx+c)^4+3\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\tan(dx+c)^3+3\,(a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)\tan(dx+c)^2+(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\tan(dx+c)}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b - 3*
a^5*b^3)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (136*a^6*b -
64*a^4*b^3 - 8*a^2*b^5 + 3*(27*a^6*b - 43*a^4*b^3 - 7*a^2*b^5 - b^7)*tan(d*
x + c)^6 + 3*(11*a^7 + 5*a^5*b^2 - 7*a^3*b^4 - a*b^6)*tan(d*x + c)^5 + 8*(4
1*a^6*b - 31*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^4 + 8*(5*a^7 - 4*a^5*b^2
 - 11*a^3*b^4 - 2*a*b^6)*tan(d*x + c)^3 + 3*(125*a^6*b - 69*a^4*b^3 - a^2*b
^5 + b^7)*tan(d*x + c)^2 + (15*a^7 - 23*a^5*b^2 - 43*a^3*b^4 - 5*a*b^6)*tan
```

```
(d*x + c))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + (a^8*b + 4*a^
6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^7 + (a^9 + 4*a^7*b^2 + 6*
a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^6 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^5 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*tan(d*x + c)^4 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*tan(d*x + c)^3 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*tan(d*x + c)^2 + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan
(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(289) = 578$.
time = 0.42, size = 619, normalized size = 2.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^7 -
 2*(19*a^8*b + 68*a^6*b^3 + 90*a^4*b^5 + 52*a^2*b^7 + 11*b^9)*cos(d*x + c)^
5 + (85*a^8*b + 224*a^6*b^3 + 210*a^4*b^5 + 88*a^2*b^7 + 17*b^9)*cos(d*x +
c)^3 - (17*a^8*b + 72*a^6*b^3 + 120*a^4*b^5 + 20*a^2*b^7 + 3*b^9 + 3*(5*a^9
 - 80*a^7*b^2 + 50*a^5*b^4 + 8*a^3*b^6 + a*b^8)*d*x)*cos(d*x + c) - 48*((a^
8*b - 3*a^6*b^3)*cos(d*x + c) + (a^7*b^2 - 3*a^5*b^4)*sin(d*x + c))*log(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (98*a^7*
b^2 + 24*a^5*b^4 - 30*a^3*b^6 - 4*a*b^8 - 8*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cos(d*x + c)^6 + 2*(13*a^9 + 44*a^7*b^2 + 54*a^5*b^4 + 2
8*a^3*b^6 + 5*a*b^8)*cos(d*x + c)^4 + 3*(5*a^8*b - 80*a^6*b^3 + 50*a^4*b^5
+ 8*a^2*b^7 + b^9)*d*x - 3*(11*a^9 + 16*a^7*b^2 - 2*a^5*b^4 - 8*a^3*b^6 - a
*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^
5*b^6 + 5*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^10*b + 5*a^8*b^3 + 10*a^6*b
^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*d*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(289) =$ 578.

time = 0.54, size = 735, normalized size = 2.47

$$\frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{96\,(a^7b^2-3\,a^5b^4)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}-\frac{48\,(2\,a^7b^2\tan(dx+c)-6\,a^5b^4\tan(dx+c)+3\,a^8b-5\,a^6b^3)}{(a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10})(b\tan(dx+c)+a)}+\frac{88\,a^7b\tan(dx+c)^6-264\,a^5b^3\tan(dx+c)^6-33\,a^8\tan(dx+c)^5+96\,a^6b^2\tan(dx+c)^5+150\,a^4b^4\tan(dx+c)^5+24\,a^2b^6\tan(dx+c)^5+3\,b^8\tan(dx+c)^5+120\,a^7b\tan(dx+c)^4-936\,a^5b^3\tan(dx+c)^4-40\,a^8\tan(dx+c)^3+208\,a^6b^2\tan(dx+c)^3+240\,a^4b^4\tan(dx+c)^3-16\,a^2b^6\tan(dx+c)^3-8\,b^8\tan(dx+c)^3+48\,a^7b\tan(dx+c)^2-912\,a^5b^3\tan(dx+c)^2+120\,a^3b^5\tan(dx+c)^2+24\,ab^7\tan(dx+c)^2-15\,a^8\tan(dx+c)+96\,a^6b^2\tan(dx+c)+90\,a^4b^4\tan(dx+c)-24\,a^2b^6\tan(dx+c)-3\,b^8\tan(dx+c)-288\,a^5b^3+72\,a^3b^5+8\,ab^7}{(a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10})\left(\tan(dx+c)^2+1\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*
a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) + 96*(a^7*b^2 - 3*a^5*b^4)*log(abs(b*tan(d*x + c) + a
))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) - 48*(
2*a^7*b^2*tan(d*x + c) - 6*a^5*b^4*tan(d*x + c) + 3*a^8*b - 5*a^6*b^3)/((a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(d*x + c
) + a)) + (88*a^7*b*tan(d*x + c)^6 - 264*a^5*b^3*tan(d*x + c)^6 - 33*a^8*ta
n(d*x + c)^5 + 96*a^6*b^2*tan(d*x + c)^5 + 150*a^4*b^4*tan(d*x + c)^5 + 24*
a^2*b^6*tan(d*x + c)^5 + 3*b^8*tan(d*x + c)^5 + 120*a^7*b*tan(d*x + c)^4 -
936*a^5*b^3*tan(d*x + c)^4 - 40*a^8*tan(d*x + c)^3 + 208*a^6*b^2*tan(d*x +
c)^3 + 240*a^4*b^4*tan(d*x + c)^3 - 16*a^2*b^6*tan(d*x + c)^3 - 8*b^8*tan(d
*x + c)^3 + 48*a^7*b*tan(d*x + c)^2 - 912*a^5*b^3*tan(d*x + c)^2 + 120*a^3*
b^5*tan(d*x + c)^2 + 24*a*b^7*tan(d*x + c)^2 - 15*a^8*tan(d*x + c) + 96*a^6
*b^2*tan(d*x + c) + 90*a^4*b^4*tan(d*x + c) - 24*a^2*b^6*tan(d*x + c) - 3*b
^8*tan(d*x + c) - 288*a^5*b^3 + 72*a^3*b^5 + 8*a*b^7)/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)^3))/d
```

Mupad [B]
time = 5.54, size = 757, normalized size = 2.55

$$\frac{\ln(a+b\tan(c+dx))\left(\frac{2ab}{(a^2+b^2)^2}-\frac{12ab^3}{(a^2+b^2)^3}+\frac{18ab^5}{(a^2+b^2)^4}-\frac{8ab^7}{(a^2+b^2)^5}\right)}{d}+\frac{\frac{\tan(c+dx)^3\,(-5a^5+9a^3b^2+2ab^4)}{6\,(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{\tan(c+dx)^5\,(-11a^5+6a^3b^2+ab^4)}{16\,(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{\tan(c+dx)^6\,(-27a^6b+43a^4b^3+7a^2b^5+b^7)}{16\,(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}+\frac{\tan(c+dx)\,(-15a^5+38a^3b^2+5ab^4)}{48\,(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{a\,(-17a^5b+8a^3b^3+ab^5)}{6\,(a^2+b^2)\,(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{\tan(c+dx)^4\,(41a^6b-31a^4b^3+a^2b^5+b^7)}{6\,(a^2+b^2)\,(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{\tan(c+dx)^2\,(125a^6b-69a^4b^3-a^2b^5+b^7)}{16\,(a^2+b^2)\,(a^6+3a^4b^2+3a^2b^4+b^6)}}{d\,\left(b\tan(c+dx)^7+a\tan(c+dx)^6+3b\tan(c+dx)^5+3a\tan(c+dx)^4+3b\tan(c+dx)^3+3a\tan(c+dx)^2+b\tan(c+dx)+a\right)}+\frac{\ln(\tan(c+dx)+1\mathrm{i})\,(a^3\,5\mathrm{i}-7a^2b+ab^2\,5\mathrm{i}+b^3)}{32\,d\,(a^5-a^4b\,5\mathrm{i}-10a^3b^2+a^2b^3\,10\mathrm{i}+5ab^4-b^5\,1\mathrm{i})}-\frac{\ln(\tan(c+dx)-\mathrm{i})\,(a^3\,5\mathrm{i}+7a^2b+ab^2\,5\mathrm{i}-b^3)}{32\,d\,(a^5+a^4b\,5\mathrm{i}-10a^3b^2-a^2b^3\,10\mathrm{i}+5ab^4+b^5\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (12*a*b^3)/(a^2 + b^2)^3
+ (18*a*b^5)/(a^2 + b^2)^4 - (8*a*b^7)/(a^2 + b^2)^5))/d + ((tan(c + d*x)^3
*(2*a*b^4 - 5*a^5 + 9*a^3*b^2))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
tan(c + d*x)^5*(a*b^4 - 11*a^5 + 6*a^3*b^2))/(16*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^6*(b^7 - 27*a^6*b + 7*a^2*b^5 + 43*a^4*b^3))/(16
*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a*b^4
- 15*a^5 + 38*a^3*b^2))/(48*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*(a*b^
5 - 17*a^5*b + 8*a^3*b^3))/(6*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)) - (tan(c + d*x)^4*(41*a^6*b + b^7 + a^2*b^5 - 31*a^4*b^3))/(6*(a^2 + b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c + d*x)^2*(125*a^6*b + b^7
- a^2*b^5 - 69*a^4*b^3))/(16*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))/(d*(a + b*tan(c + d*x) + 3*a*tan(c + d*x)^2 + 3*a*tan(c + d*x)^4 + a*ta
```

```
n(c + d*x)^6 + 3*b*tan(c + d*x)^3 + 3*b*tan(c + d*x)^5 + b*tan(c + d*x)^7))
 + (log(tan(c + d*x) + 1i)*(a*b^2*5i - 7*a^2*b + a^3*5i + b^3))/(32*d*(5*a*
b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)) - (log(tan(c + d
*x) - 1i)*(a*b^2*5i + 7*a^2*b + a^3*5i - b^3))/(32*d*(5*a*b^4 + a^4*b*5i +
a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2))
```

3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)\,x}{8\,(a^2+b^2)^4}+\frac{2a^3b(a^2-2b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{a^4b}{(a^2+b^2)^3\,d(a+b\tan(c+$$

```
[Out] 1/8*(3*a^6-33*a^4*b^2+13*a^2*b^4+b^6)*x/(a^2+b^2)^4+2*a^3*b*(a^2-2*b^2)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d-1/8*cos(d*x+
c)^2*(16*a^3*b+(5*a^4-12*a^2*b^2-b^4)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A]
time = 0.45, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\frac{\cos^4(c+dx)\left((a^2-b^2)\tan(c+dx)+2ab\right)}{4d\,(a^2+b^2)^2}-\frac{a^4b}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{2a^3b(a^2-2b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{x(3a^6-33a^4b^2+13a^2b^4+b^6)}{8\,(a^2+b^2)^4}-\frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx)\right)}{8d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a
^2 - 2*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*
b)/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(2*a*b + (a^2 -
 b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(16*a^3*b + (5*a
^4 - 12*a^2*b^2 - b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^2(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2} - \frac{2ab^4\left(3a^2+b^2\right)x}{\left(a^2+b^2\right)^2} -}{(a+x)^2(b^2+x}\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{2a^3b(a^2-2b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(}{}$$

$$= \frac{2a^3b(a^2-2b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(}{}$$

$$= \frac{\left(3a^6-33a^4b^2+13a^2b^4+b^6\right)x}{8\left(a^2+b^2\right)^4} + \frac{2a^3b(a^2-2b^2)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{2a^3b(a^2-}{}$$

Mathematica [A]
time = 4.19, size = 373, normalized size = 1.72

$$\frac{b\left(\frac{4(a^2+b^2)(-2a^4+3a^2b^2+b^4)\text{ArcTan}(\tan(c+dx))}{b} - 16a^3(a^2+b^2)\cos^2(c+dx) + 4a(a^2+b^2)^2\cos^4(c+dx) - 4a^3\left(2a^2-4b^2+\frac{-a^3+5ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + 16a^3(a^2-2b^2)\log(a+b\tan(c+dx)) - 4a^3\left(2a^2-4b^2+\frac{a^3-5ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right) + \frac{2(a^2-b^2)(a^2+b^2)^2\cos^3(c+dx)\sin(c+dx)}{b} + \frac{2(a^2+b^2)(-2a^4+3a^2b^2+b^4)\sin(2(c+dx))}{b} + \frac{3(a^2-b^2)(a^2+b^2)^2(2\text{ArcTan}(\tan(c+dx))+\sin(2(c+dx)))}{2b} - \frac{8a^4(a^2+b^2)}{a+b\tan(c+dx)}\right)}{8\left(a^2+b^2\right)^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*
a^3*(a^2 + b^2)*Cos[c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(
2*a^2 - 4*b^2 + (-a^3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]] + 16*a^3*(a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 +
(a^3 - 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^
2)*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(a^2 + b^2)*(-2*a^4 +
3*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(a^2 + b^2)^2*(2*ArcT
an[Tan[c + d*x]] + Sin[2*(c + d*x)]))/(2*b) - (8*a^4*(a^2 + b^2))/(a + b*Ta
n[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

Maple [A]
time = 0.38, size = 269, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{b\,a^{4}}{\left(a^{2}+b^{2}\right)^{3}\left(a+b\tan\left(dx+c\right)\right)}+\frac{2a^{3}b\left(a^{2}-2b^{2}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{4}}+\frac{\left(-\frac{5}{8}a^{6}+\frac{7}{8}a^{4}b^{2}+\frac{13}{8}a^{2}b^{4}+\frac{1}{8}b^{6}\right)\left(\tan^{3}\left(dx+c\right)\right)+\left(-2a^{5}b-2a^{3}b^{3}\right.}{}}{}$
default	$\frac{-\frac{b\,a^{4}}{\left(a^{2}+b^{2}\right)^{3}\left(a+b\tan\left(dx+c\right)\right)}+\frac{2a^{3}b\left(a^{2}-2b^{2}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{4}}+\frac{\left(-\frac{5}{8}a^{6}+\frac{7}{8}a^{4}b^{2}+\frac{13}{8}a^{2}b^{4}+\frac{1}{8}b^{6}\right)\left(\tan^{3}\left(dx+c\right)\right)+\left(-2a^{5}b-2a^{3}b^{3}\right.}{}}{}$
risch	$-\frac{ixab}{2\left(4ia^{3}b-4ia\,b^{3}-a^{4}+6a^{2}b^{2}-b^{4}\right)}-\frac{3x\,a^{2}}{8\left(4ia^{3}b-4ia\,b^{3}-a^{4}+6a^{2}b^{2}-b^{4}\right)}-\frac{x\,b^{2}}{8\left(4ia^{3}b-4ia\,b^{3}-a^{4}+6a^{2}b^{2}-b^{4}\right)}-\frac{ie}{64(-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*a^4/(a^2+b^2)^3/(a+b*tan(d*x+c))+2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4*ln
(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(((-5/8*a^6+7/8*a^4*b^2+13/8*a^2*b^4+1/8*b^6
)*tan(d*x+c)^3+(-2*a^5*b-2*a^3*b^3)*tan(d*x+c)^2+(-3/8*a^6+9/8*a^4*b^2+11/8
*a^2*b^4-1/8*b^6)*tan(d*x+c)-3/2*a^5*b-a^3*b^3+1/2*a*b^5)/(1+tan(d*x+c)^2)^
2+1/16*(-16*a^5*b+32*a^3*b^3)*ln(1+tan(d*x+c)^2)+1/8*(3*a^6-33*a^4*b^2+13*a
^2*b^4+b^6)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(211) = 422.
time = 0.55, size = 507, normalized size = 2.34

$$\frac{\frac{\left(3a^{6}-33a^{4}b^{2}+13a^{2}b^{4}+b^{6}\right)\left(dx+c\right)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}+\frac{16\left(a^{5}b-2a^{3}b^{3}\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{8\left(a^{5}b-2a^{3}b^{3}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{20a^{4}b-4a^{2}b^{3}+\left(13a^{4}b-12a^{2}b^{3}-b^{5}\right)\tan\left(dx+c\right)^{4}+\left(5a^{5}+4a^{3}b^{2}-ab^{4}\right)\tan\left(dx+c\right)^{3}+\left(35a^{4}b-12a^{2}b^{3}+b^{5}\right)\tan\left(dx+c\right)^{2}+3\left(a^{5}-ab^{4}\right)\tan\left(dx+c\right)}{a^{7}+3a^{5}b^{2}+3a^{3}b^{4}+ab^{6}+\left(a^{6}b+3a^{4}b^{3}+3a^{2}b^{5}+b^{7}\right)\tan\left(dx+c\right)^{5}+\left(a^{7}+3a^{5}b^{2}+3a^{3}b^{4}+ab^{6}\right)\tan\left(dx+c\right)^{4}+2\left(a^{6}b+3a^{4}b^{3}+3a^{2}b^{5}+b^{7}\right)\tan\left(dx+c\right)^{3}+2\left(a^{7}+3a^{5}b^{2}+3a^{3}b^{4}+ab^{6}\right)\tan\left(dx+c\right)^{2}+\left(a^{6}b+3a^{4}b^{3}+3a^{2}b^{5}+b^{7}\right)\tan\left(dx+c\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) + a
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*l
og(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (2
0*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (5*a^5
 + 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*tan(d*
x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^
2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(211) = 422$.
time = 0.40, size = 444, normalized size = 2.05

$$\frac{4\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos{(dx + c)}^5 - 6\,(3\,a^6b + 7\,a^4b^3 + 5\,a^2b^5 + b^7)\cos{(dx + c)}^3 + (3\,a^6b + 8\,a^4b^3 + 23\,a^2b^5 + 2\,b^7 + 2\,(3\,a^7 - 33\,a^5b^2 + 13\,a^3b^4 + ab^6)dx)\cos{(dx + c)} + 16\,((a^6b - 2\,a^4b^3)\cos{(dx + c)} + (a^5b^2 - 2\,a^3b^4)\sin{(dx + c)})\log{(2\,ab\cos{(dx + c)}\sin{(dx + c)} + (a^2 - b^2)\cos{(dx + c)}^2 + b^2)} + (29\,a^5b^2 + 10\,a^3b^4 - 3\,ab^6 + 4\,(a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos{(dx + c)}^4 + 2\,(3\,a^6b - 33\,a^4b^3 + 13\,a^2b^5 + b^7)dx - 2\,(5\,a^7 + 9\,a^5b^2 + 3\,a^3b^4 - ab^6)\cos{(dx + c)}^2)\sin{(dx + c)}}{16\,((a^9 + 4\,a^7b^2 + 6\,a^5b^4 + 4\,a^3b^6 + ab^8)d\cos{(dx + c)} + (a^8b + 4\,a^6b^3 + 6\,a^4b^5 + 4\,a^2b^7 + b^9)d\sin{(dx + c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 6*(3*a^6*b +
 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*a^
2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(d*x +
c) + 16*((a^6*b - 2*a^4*b^3)*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4)*sin(d*x +
 c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(d*x + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d*x - 2*
(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a
^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(211) = 422$.
time = 0.55, size = 513, normalized size = 2.36

$$\frac{\frac{(3\,a^6-33\,a^4b^2+13\,a^2b^4+b^6)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{8\,(a^5b-2\,a^3b^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{16\,(a^5b^2-2\,a^3b^4)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} - \frac{8\,(2\,a^5b^2\tan(dx+c)-4\,a^3b^4\tan(dx+c)+3\,a^6b-3\,a^4b^3)}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)} + \frac{12\,a^5b\tan(dx+c)^4-24\,a^3b^3\tan(dx+c)^4-5\,a^6\tan(dx+c)^3+7\,a^4b^2\tan(dx+c)^3+13\,a^2b^4\tan(dx+c)^3+b^6\tan(dx+c)^3+8\,a^5b\tan(dx+c)^2-64\,a^3b^3\tan(dx+c)^2-3\,a^6\tan(dx+c)+9\,a^4b^2\tan(dx+c)+11\,a^2b^4\tan(dx+c)-b^6\tan(dx+c)-32\,a^3b^3+4\,ab^5}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b^4)
*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 +
b^9) - 8*(2*a^5*b^2*tan(d*x + c) - 4*a^3*b^4*tan(d*x + c) + 3*a^6*b - 3*a^4
```

```
*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)
) + (12*a^5*b*tan(d*x + c)^4 - 24*a^3*b^3*tan(d*x + c)^4 - 5*a^6*tan(d*x +
c)^3 + 7*a^4*b^2*tan(d*x + c)^3 + 13*a^2*b^4*tan(d*x + c)^3 + b^6*tan(d*x +
 c)^3 + 8*a^5*b*tan(d*x + c)^2 - 64*a^3*b^3*tan(d*x + c)^2 - 3*a^6*tan(d*x
+ c) + 9*a^4*b^2*tan(d*x + c) + 11*a^2*b^4*tan(d*x + c) - b^6*tan(d*x + c)
- 32*a^3*b^3 + 4*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(t
an(d*x + c)^2 + 1)^2))/d
```

Mupad [B]
time = 4.82, size = 481, normalized size = 2.22

$$\frac{\frac{\tan(c+d\,x)^3\,\left(a\,b^2-5\,a^3\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)^4\,\left(-13\,a^4\,b+12\,a^2\,b^3+b^5\right)}{8\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{3\,\tan(c+d\,x)\,\left(a\,b^2-a^3\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\tan(c+d\,x)^2\,\left(35\,a^4\,b-12\,a^2\,b^3+b^5\right)}{8\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\,\left(a\,b^3-5\,a^3\,b\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(b\,\tan\left(c+d\,x\right)^5+a\,\tan\left(c+d\,x\right)^4+2\,b\,\tan\left(c+d\,x\right)^3+2\,a\,\tan\left(c+d\,x\right)^2+b\,\tan\left(c+d\,x\right)+a\right)}+\frac{\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{8\,a\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{6\,a\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,a^2-a\,b\,4{}\mathrm{i}+b^2\right)}{16\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(3\,a^2+a\,b\,4{}\mathrm{i}+b^2\right)}{16\,d\,\left(a^4\,1{}\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}-4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((tan(c + d*x)^3*(a*b^2 - 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*
x)^4*(b^5 - 13*a^4*b + 12*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (3*tan(c + d*x)*(a*b^2 - a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c + d*
x)^2*(35*a^4*b + b^5 - 12*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
 + (a*(a*b^3 - 5*a^3*b))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + b
*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*tan(c + d*x)^3
+ b*tan(c + d*x)^5)) + (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (8
*a*b^3)/(a^2 + b^2)^3 + (6*a*b^5)/(a^2 + b^2)^4))/d + (log(tan(c + d*x) - 1
i)*(3*a^2 - a*b*4i + b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2
*b^2*6i)) - (log(tan(c + d*x) + 1i)*(a*b*4i + 3*a^2 + b^2))/(16*d*(4*a^3*b
- 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3}+\frac{2ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{a^2b}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}-\frac{\cos^2(c}{}$$

```
[Out] 1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin
(d*x+c))/(a^2+b^2)^3/d-a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^
2*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{a^2b}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\,((a^2-b^2)\tan(c+dx)+2ab)}{2d\,(a^2+b^2)^2}+\frac{2ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{x(a^4-6a^2b^2+b^4)}{2\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*
(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(2*a*b + (a^2 - b^2)*Tan[c + d*x]))
/(2*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^2(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^2x}{a^2+b^2}+\frac{b^2 \cdots}{\cdots}}{(a+x)^2(b^2+x^2)} \cdots\right.}{2b \cdots}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)^2}+\frac{4a \cdots}{(a^2 \cdots}\right.\right.}{\cdots}$$

$$= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(\cdots}{\cdots}$$

$$= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(\cdots}{\cdots}$$

$$= \frac{(a^4-6a^2b^2+b^4)\,x}{2\left(a^2+b^2\right)^3} + \frac{2ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{2ab(a^2-b^2)\log(a+b\,\text{t}\cdots}{(a^2+b^2)^3 d}$$

Mathematica [A]
time = 3.52, size = 246, normalized size = 1.66

$$-\frac{b\left(\frac{(a^2-b^2)(a^2+b^2)\text{ArcTan}(\tan(c+dx))}{b}+2a(a^2+b^2)\cos^2(c+dx)+a\left(2a^2-2b^2+\frac{-a^3+3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-4a(a-b)(a+b)\log(a+b\tan(c+dx))+a\left(2a^2-2b^2+\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+\frac{(a-b)(a+b)(a^2+b^2)\sin(2(c+dx))}{2b}+\frac{2a^2(a^2+b^2)}{a+b\tan(c+dx)}\right)}{2\left(a^2+b^2\right)^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2)
*Cos[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[
-b^2] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] + a*(
2*a^2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]
] + ((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^
2))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^3*d)
```

Maple [A]
time = 0.41, size = 171, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{a^{2}b}{\left(a^{2}+b^{2}\right)^{2}\left(a+b\tan(dx+c)\right)}+\frac{2ab\left(a^{2}-b^{2}\right)\ln\left(a+b\tan(dx+c)\right)}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(-\frac{a^{4}}{2}+\frac{b^{4}}{2}\right)\tan(dx+c)-a^{3}b-a\,b^{3}}{1+\tan^{2}(dx+c)}+\frac{\left(-4a^{3}b+4a\,b^{3}\right)\ln\left(1+\tan^{2}(dx\cdots\right)}{4}\cdots}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
default	$\frac{-\frac{a^{2}b}{\left(a^{2}+b^{2}\right)^{2}\left(a+b\tan(dx+c)\right)}+\frac{2ab\left(a^{2}-b^{2}\right)\ln\left(a+b\tan(dx+c)\right)}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(-\frac{a^{4}}{2}+\frac{b^{4}}{2}\right)\tan(dx+c)-a^{3}b-a\,b^{3}}{1+\tan^{2}(dx+c)}+\frac{\left(-4a^{3}b+4a\,b^{3}\right)\ln\left(1+\tan^{2}(dx\cdots\right)}{4}\cdots}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
risch	$-\frac{ixb}{2(3ib\,a^{2}-ib^{3}-a^{3}+3b^{2}a)}-\frac{xa}{2(3ib\,a^{2}-ib^{3}-a^{3}+3b^{2}a)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-2iab+a^{2}-b^{2})d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8(2iab+a^{2}-b^{2})d}-\frac{4ia^{3}bx}{a^{6}+3a^{4}b^{2}+3a^{2}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))+2*a*b*(a^2-b^2)/(a^2+b^2)^3*ln(a+b
*tan(d*x+c))+1/(a^2+b^2)^3*(((-1/2*a^4+1/2*b^4)*tan(d*x+c)-a^3*b-a*b^3)/(1+
tan(d*x+c)^2)+1/4*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+1/2*(a^4-6*a^2*b^2+
b^4)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(144) = 288.
time = 0.52, size = 293, normalized size = 1.98

$$\frac{\frac{(a^4-6\,a^2b^2+b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{4\,(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,a^2b+(3\,a^2b-b^3)\tan(dx+c)^2+(a^3+ab^2)\tan(dx+c)}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5+2\,a^3b^2+ab^4)\tan(dx+c)^2+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ 4*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (4*a^2*b + (3*a^2*b - b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d
```

```
*x + c))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^
3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*ta
n(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.
time = 0.42, size = 292, normalized size = 1.97

$$-\frac{(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^3+(a^2b^3-b^5-(a^5-6\,a^3b^2+ab^4)dx)\cos(dx+c)-2\,((a^4b-a^2b^3)\cos(dx+c)+(a^3b^2-ab^4)\sin(dx+c))\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-\left(3\,a^3b^2+ab^4+(a^4b-6\,a^2b^3+b^5)dx-(a^5+2\,a^3b^2+ab^4)\cos(dx+c)^2\right)\sin(dx+c)}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)d\cos(dx+c)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + (a^2*b^3 - b^5 - (a^5 - 6*
a^3*b^2 + a*b^4)*d*x)*cos(d*x + c) - 2*((a^4*b - a^2*b^3)*cos(d*x + c) + (a
^3*b^2 - a*b^4)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 + b^2) - (3*a^3*b^2 + a*b^4 + (a^4*b - 6*a^2*b^3 + b^5)
*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.53, size = 263, normalized size = 1.78

$$\frac{\frac{(a^4-6\,a^2b^2+b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{4\,(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,a^2b\tan(dx+c)^2-b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+4\,a^2b}{(a^4+2\,a^2b^2+b^4)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) + 4*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7) - (3*a^2*b*tan(d*x + c)^2 - b^3*tan(d*x + c)^2 + a^3*tan
```

(d*x + c) + a*b^2*tan(d*x + c) + 4*a^2*b)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d
*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d

Mupad [B]
time = 4.06, size = 255, normalized size = 1.72

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{4\,a\,b^3}{{\left(a^2+b^2\right)}^3}\right)}{d}-\frac{\frac{{\tan(c+d\,x)}^2\,\left(3\,a^2\,b-b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}+\frac{2\,a^2\,b}{{\left(a^2+b^2\right)}^2}}{d\,\left(b\,{\tan\left(c+d\,x\right)}^3+a\,{\tan\left(c+d\,x\right)}^2+b\tan\left(c+d\,x\right)+a\right)}+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)}{4\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (4*a*b^3)/(a^2 + b^2)^3))
/d - ((tan(c + d*x)^2*(3*a^2*b - b^3))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a*tan
(c + d*x))/(2*(a^2 + b^2)) + (2*a^2*b)/(a^2 + b^2)^2)/(d*(a + b*tan(c + d*x
) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) + 1i)*(a + b*
1i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan(c + d*x) - 1i)*(a
*1i + b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
```

3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=72

$$-\frac{\cot(c+dx)}{a^2d}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^2/d-2*b*ln(tan(d*x+c))/a^3/d+2*b*ln(a+b*tan(d*x+c))/a^3/d-b/a
^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 46}

$$-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) - (2*b*Log[Tan[c + d*x]])/(a^3*d) + (2*b*Log[a + b*
Tan[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{a^2d}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{}{a^2d(a+b}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 109, normalized size = 1.51

$$\frac{-a^2\cot^2(c+dx)-ab\cot(c+dx)(1+2\log(\sin(c+dx))-2\log(a\cos(c+dx)+b\sin(c+dx)))+b^2(1-2\log(\sin(c+dx))+2\log(a\cos(c+dx)+b\sin(c+dx)))}{a^3d(b+a\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cot[c + d*x]^2) - a*b*Cot[c + d*x]*(1 + 2*Log[Sin[c + d*x]] - 2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]]) + b^2*(1 - 2*Log[Sin[c + d*x]] + 2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]]))/(a^3*d*(b + a*Cot[c + d*x]))
```

Maple [A]
time = 0.32, size = 67, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{b}{a^2(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^3}-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}}{d}$	67
default	$\frac{-\frac{b}{a^2(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^3}-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}}{d}$	67
risch	$-\frac{2i\left(2iab\,e^{2i(dx+c)}-a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}-a^2-2b^2\right)}{\left(e^{2i(dx+c)}-1\right)(ia+b)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)a^2d}-\frac{2b\ln\left(e^{2i(dx+c)}-1\right)}{a^3d}+\frac{2b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^3d}$	178

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/a^2/(a+b*tan(d*x+c))+2/a^3*b*ln(a+b*tan(d*x+c))-1/a^2/tan(d*x+c)-2/
a^3*b*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.32, size = 74, normalized size = 1.03

$$-\frac{\frac{2\,b\tan(dx+c)+a}{a^2b\tan(dx+c)^2+a^3\tan(dx+c)}-\frac{2\,b\log(b\tan(dx+c)+a)}{a^3}+\frac{2\,b\log(\tan(dx+c))}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*b*tan(d*x + c) + a)/(a^2*b*tan(d*x + c)^2 + a^3*tan(d*x + c)) - 2*b*lo
g(b*tan(d*x + c) + a)/a^3 + 2*b*log(tan(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(72) = 144.
time = 0.38, size = 293, normalized size = 4.07

$$-\frac{a^2b^2-(a^4+2\,a^2b^2)\cos(dx+c)^2-(a^3b+2\,ab^3)\cos(dx+c)\sin(dx+c)+\left(a^2b^2+b^4-(a^2b^2+b^4)\cos(dx+c)^2+(a^3b+ab^3)\cos(dx+c)\sin(dx+c)\right)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-\left(a^2b^2+b^4-(a^2b^2+b^4)\cos(dx+c)^2+(a^3b+ab^3)\cos(dx+c)\sin(dx+c)\right)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)}{(a^5b+a^3b^3)d\cos(dx+c)^2-(a^6+a^4b^2)d\cos(dx+c)\sin(dx+c)-(a^5b+a^3b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^2 - (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 - (a^3*b + 2*a*b^3)*cos(d*x +
c)*sin(d*x + c) + (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x + c)^2 + (a^3*b
+ a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x
 + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(-1/4*cos(d*x + c)^
2 + 1/4))/((a^5*b + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + a^4*b^2)*d*cos(d*x +
 c)*sin(d*x + c) - (a^5*b + a^3*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.52, size = 74, normalized size = 1.03

$$\frac{\frac{2\,b\log(|b\tan(dx+c)+a|)}{a^3}-\frac{2\,b\log(|\tan(dx+c)|)}{a^3}-\frac{2\,b\tan(dx+c)+a}{\left(b\tan(dx+c)^2+a\tan(dx+c)\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (2
*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d
```

Mupad [B]
time = 3.84, size = 79, normalized size = 1.10

$$\frac{2\,b\,\ln\left(\frac{a+b\tan(c+d\,x)}{\tan(c+d\,x)}\right)}{a^3\,d}-\frac{2\,b}{a^2\,d\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{1}{a\,d\tan\left(c+d\,x\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (2*b*log((a + b*tan(c + d*x))/tan(c + d*x)))/(a^3*d) - (2*b)/(a^2*d*(a + b*
tan(c + d*x))) - 1/(a*d*tan(c + d*x)*(a + b*tan(c + d*x)))
```

3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=140

$$-\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}+\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d}+\frac{2b(a^2+2b^2)\log(a+}{a^5d}$$

```
[Out] -(a^2+3*b^2)*cot(d*x+c)/a^4/d+b*cot(d*x+c)^2/a^3/d-1/3*cot(d*x+c)^3/a^2/d-2
*b*(a^2+2*b^2)*ln(tan(d*x+c))/a^5/d+2*b*(a^2+2*b^2)*ln(a+b*tan(d*x+c))/a^5/
d-b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d}+\frac{2b(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5d}-\frac{b(a^2+b^2)}{a^4d(a+b\tan(c+dx))}-\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[
c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b
*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a
 + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^2x^4} - \frac{2b^2}{a^3x^3} + \frac{a^2+3b^2}{a^4x^2} - \frac{2(a^2+2b^2)}{a^5x} + \frac{a^2+b^2}{a^4(a+x)^2} + \frac{2(a^2+2b^2)}{a^5(a+x)}\right)\,dx, x, b\tan\right.}{d}$$

$$= -\frac{(a^2+3b^2)\cot(c+dx)}{a^4d} + \frac{b\cot^2(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^2d} - \frac{2b(a^2+2b^2)\log(\mathrm{t}}{a^5d}$$

Mathematica [A]
time = 2.83, size = 244, normalized size = 1.74

$$\frac{-\cot^2(c+dx)\left(2a^4+9a^2b^2+a^4\csc^2(c+dx)\right)+3b^2\left(a^2+b^2+a^2\csc^2(c+dx)-2(a^2+2b^2)\log(\sin(c+dx))+2a^2\log(a\cos(c+dx)+b\sin(c+dx))+4b^2\log(a\cos(c+dx)+b\sin(c+dx))\right)+ab\cot(c+dx)\left(-2a^2-9b^2+2a^2\csc^2(c+dx)-6(a^2+2b^2)\log(\sin(c+dx))+6a^2\log(a\cos(c+dx)+b\sin(c+dx))+12b^2\log(a\cos(c+dx)+b\sin(c+dx))\right)}{3a^5d(b+a\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(Cot[c + d*x]^2*(2*a^4 + 9*a^2*b^2 + a^4*Csc[c + d*x]^2)) + 3*b^2*(a^2 +
b^2 + a^2*Csc[c + d*x]^2 - 2*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 2*a^2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]] + 4*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
) + a*b*Cot[c + d*x]*(-2*a^2 - 9*b^2 + 2*a^2*Csc[c + d*x]^2 - 6*(a^2 + 2*b^
2)*Log[Sin[c + d*x]] + 6*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 12*b^2*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]]))/(3*a^5*d*(b + a*Cot[c + d*x]))
```

Maple [A]
time = 0.37, size = 127, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{(a^2+b^2)b}{a^4(a+b\tan(dx+c))}+\frac{2b(a^2+2b^2)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b(a^2+2b^2)\ln(\tan(dx}{a^5}}{d}$
default	$\frac{-\frac{(a^2+b^2)b}{a^4(a+b\tan(dx+c))}+\frac{2b(a^2+2b^2)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b(a^2+2b^2)\ln(\tan(dx}{a^5}}{d}$
risch	$-\frac{4i\left(-3ia^3e^{4i(dx+c)}+6iab^2-3a^2be^{4i(dx+c)}+a^2be^{2i(dx+c)}+6b^3e^{6i(dx+c)}-18b^3e^{4i(dx+c)}+3a^2be^{6i(dx+c)}-2ia^3e^{2i(dx+}\right.}{3\left(e^{2i(dx+c)}-1\right)^3\left(be^{2i(dx+c)}+iae^{2i(dx+c)}-b+ia\right)e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^2+b^2)*b/a^4/(a+b*tan(d*x+c))+2*b*(a^2+2*b^2)/a^5*ln(a+b*tan(d*x+c
))-1/3/a^2/tan(d*x+c)^3-(a^2+3*b^2)/a^4/tan(d*x+c)+1/a^3*b/tan(d*x+c)^2-2*b
*(a^2+2*b^2)/a^5*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.31, size = 144, normalized size = 1.03

$$\frac{\frac{2\,a^2b\tan(dx+c)-6\,(a^2b+2\,b^3)\,\tan(dx+c)^3-a^3-3\,(a^3+2\,ab^2)\,\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6\,(a^2b+2\,b^3)\,\log(b\tan(dx+c)+a)}{a^5}-\frac{6\,(a^2b+2\,b^3)\,\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((2*a^2*b*tan(d*x + c) - 6*(a^2*b + 2*b^3)*tan(d*x + c)^3 - a^3 - 3*(a^
3 + 2*a*b^2)*tan(d*x + c)^2)/(a^4*b*tan(d*x + c)^4 + a^5*tan(d*x + c)^3) +
6*(a^2*b + 2*b^3)*log(b*tan(d*x + c) + a)/a^5 - 6*(a^2*b + 2*b^3)*log(tan(d
*x + c))/a^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(138) = 276.
time = 0.38, size = 442, normalized size = 3.16

$$\frac{2\,(a^4+6\,a^2b^2)\cos(dx+c)^4+6\,a^2b^2-3\,(a^4+6\,a^2b^2)\cos(dx+c)^2+3\,\big((a^2b^2+2\,b^4)\cos(dx+c)^4+a^2b^2+2\,b^4-2\,(a^2b^2+2\,b^4)\cos(dx+c)^2-\big((a^3b+2\,ab^3)\cos(dx+c)^3-(a^3b+2\,ab^3)\cos(dx+c)\big)\sin(dx+c)\big)\log\big(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\big)-3\,\big((a^2b^2+2\,b^4)\cos(dx+c)^4+a^2b^2+2\,b^4-2\,(a^2b^2+2\,b^4)\cos(dx+c)^2-\big((a^3b+2\,ab^3)\cos(dx+c)^3-(a^3b+2\,ab^3)\cos(dx+c)\big)\sin(dx+c)\big)\log\big(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\big)-2\,\big(6\,ab^3\cos(dx+c)-(a^3b+6\,ab^3)\cos(dx+c)^3\big)\sin(dx+c)}{3\,\big(a^5bd\cos(dx+c)^4-2\,a^5bd\cos(dx+c)^2+a^5bd-\big(a^6d\cos(dx+c)^3-a^6d\cos(dx+c)\big)\sin(dx+c)\big)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^4 + 6*a^2*b^2)*cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*c
os(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(
a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)^3 - (a^3*
b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4
 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3
)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(-1/4*c
os(d*x + c)^2 + 1/4) - 2*(6*a*b^3*cos(d*x + c) - (a^3*b + 6*a*b^3)*cos(d*x
+ c)^3)*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 +
a^5*b*d - (a^6*d*cos(d*x + c)^3 - a^6*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 203, normalized size = 1.45

$$-\frac{\frac{6\,(a^2b+2\,b^3)\log(|\tan(dx+c)|)}{a^5}-\frac{6\,(a^2b^2+2\,b^4)\log(|b\tan(dx+c)+a|)}{a^5b}+\frac{3\,(2\,a^2b^2\tan(dx+c)+4\,b^4\tan(dx+c)+3\,a^3b+5\,ab^3)}{(b\tan(dx+c)+a)a^5}-\frac{11\,a^2b\tan(dx+c)^3+22\,b^3\tan(dx+c)^3-3\,a^3\tan(dx+c)^2-9\,ab^2\tan(dx+c)^2+3\,a^2b\tan(dx+c)-a^3}{a^5\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(a^2*b + 2*b^3)*log(abs(tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*lo
g(abs(b*tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*tan(d*x + c) + 4*b^4*tan(
d*x + c) + 3*a^3*b + 5*a*b^3)/((b*tan(d*x + c) + a)*a^5) - (11*a^2*b*tan(d*
x + c)^3 + 22*b^3*tan(d*x + c)^3 - 3*a^3*tan(d*x + c)^2 - 9*a*b^2*tan(d*x +
 c)^2 + 3*a^2*b*tan(d*x + c) - a^3)/(a^5*tan(d*x + c)^3))/d
```

Mupad [B]
time = 3.91, size = 150, normalized size = 1.07

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,(a^2+2\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(2\,a^2\,b+4\,b^3)}\right)\,(a^2+2\,b^2)}{a^5\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+2\,b^2)}{a^3}-\frac{2\,b\tan(c+d\,x)}{3\,a^2}+\frac{2\,b\tan(c+d\,x)^3\,(a^2+2\,b^2)}{a^4}}{d\,\left(b\tan\left(c+d\,x\right)^4+a\tan\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 2*b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^2*b + 4*b^3))
)*(a^2 + 2*b^2))/(a^5*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 2*b^2))/a^3 -
(2*b*tan(c + d*x))/(3*a^2) + (2*b*tan(c + d*x)^3*(a^2 + 2*b^2))/a^4)/(d*(a*
tan(c + d*x)^3 + b*tan(c + d*x)^4))
```

3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d}+\frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}+\frac{b\cot^4(c+dx)}{2a^3d}-\frac{\cot^5}{5}$$

```
[Out] -(a^2+b^2)*(a^2+5*b^2)*cot(d*x+c)/a^6/d+2*b*(a^2+b^2)*cot(d*x+c)^2/a^5/d-1/
3*(2*a^2+3*b^2)*cot(d*x+c)^3/a^4/d+1/2*b*cot(d*x+c)^4/a^3/d-1/5*cot(d*x+c)^
5/a^2/d-2*b*(a^2+b^2)*(a^2+3*b^2)*ln(tan(d*x+c))/a^7/d+2*b*(a^2+b^2)*(a^2+3
*b^2)*ln(a+b*tan(d*x+c))/a^7/d-b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{b\cot^4(c+dx)}{2a^3d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2b(a^2+b^2)(a^2+3b^2)\log(\tan(c+dx))}{a^7d}+\frac{2b(a^2+b^2)(a^2+3b^2)\log(a+b\tan(c+dx))}{a^7d}-\frac{b(a^2+b^2)^2}{a^6d(a+b\tan(c+dx))}-\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d}+\frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[
c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c
 + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3
*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b
*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{2a^2b^2+3b^4}{a^4x^4} - \frac{4b^2(a^2+b^2)}{a^5x^3} + \frac{a^4+6a^2b^2+5b^4}{a^6x^2} - \frac{2(a^4+4a^2b^2+3b^4)}{a^7x}\right.\right.}{d}$$
$$= -\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d} + \frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d} - \frac{(2a^2+3b^2)\text{co}}{3a^4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 589 vs. 2(219) = 438.
time = 6.26, size = 589, normalized size = 2.69

$$-\frac{\csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{5a^2d(a+b\tan(c+dx))^2} + \frac{(-8a^4\cos(c+dx)-75a^2b^2\cos(c+dx)-75b^4\cos(c+dx))\csc(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{15a^6d(a+b\tan(c+dx))^2} + \frac{b(a^2+2b^2)\csc^2(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^5d(a+b\tan(c+dx))^2} + \frac{(-4a^2\cos(c+dx)-15b^2\cos(c+dx))\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{15a^4d(a+b\tan(c+dx))^2} + \frac{b\csc^4(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2a^3d(a+b\tan(c+dx))^2} - \frac{2(a^4b+4a^2b^3+3b^5)\log(\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2} + \frac{2(a^4b+4a^2b^3+3b^5)\log(a\cos(c+dx)+b\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2} + \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))(a^4b^2\sin(c+dx)+2a^2b^4\sin(c+dx)+b^6\sin(c+dx))}{a^7d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/5*(Csc[c + d*x]^5*Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2
*d*(a + b*Tan[c + d*x])^2) + ((-8*a^4*Cos[c + d*x] - 75*a^2*b^2*Cos[c + d*x
] - 75*b^4*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2)/(15*a^6*d*(a + b*Tan[c + d*x])^2) + (b*(a^2 + 2*b^2)*Csc[c +
 d*x]^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^5*d*(a + b*T
an[c + d*x])^2) + ((-4*a^2*Cos[c + d*x] - 15*b^2*Cos[c + d*x])*Csc[c + d*x]
^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(15*a^4*d*(a + b*Tan
[c + d*x])^2) + (b*Csc[c + d*x]^4*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(2*a^3*d*(a + b*Tan[c + d*x])^2) - (2*(a^4*b + 4*a^2*b^3 + 3*b^5
)*Log[Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^
7*d*(a + b*Tan[c + d*x])^2) + (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(a^7*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])*(a^4*b^2*Sin[c + d*x] + 2*a^2*b^4*Sin[c + d*x] + b^6*Sin[c + d*x]))
/(a^7*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.37, size = 205, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{(a^4+2a^2b^2+b^4)b}{a^6(a+b\tan(dx+c))} + \frac{2b(a^4+4a^2b^2+3b^4)\ln(a+b\tan(dx+c))}{a^7} - \frac{1}{5a^2\tan(dx+c)^5} - \frac{2a^2+3b^2}{3a^4\tan(dx+c)^3} - \frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)} + \frac{b}{2a^3\tan(dx+}}{d}$
default	$\frac{-\frac{(a^4+2a^2b^2+b^4)b}{a^6(a+b\tan(dx+c))} + \frac{2b(a^4+4a^2b^2+3b^4)\ln(a+b\tan(dx+c))}{a^7} - \frac{1}{5a^2\tan(dx+c)^5} - \frac{2a^2+3b^2}{3a^4\tan(dx+c)^3} - \frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)} + \frac{b}{2a^3\tan(dx+}}{d}$

risch	$-\frac{4i\left(-4a^4b-45a^2b^3-45b^5+40a^4b\,\mathrm{e}^{6i(dx+c)}-420a^2b^3\mathrm{e}^{4i(dx+c)}+450a^2b^3\mathrm{e}^{6i(dx+c)}+40ia^5\mathrm{e}^{6i(dx+c)}+20ia^5\mathrm{e}^{4i(dx+c)}+45i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^4+2*a^2*b^2+b^4)*b/a^6/(a+b*tan(d*x+c))+2*b*(a^4+4*a^2*b^2+3*b^4)/
a^7*ln(a+b*tan(d*x+c))-1/5/a^2/tan(d*x+c)^5-1/3*(2*a^2+3*b^2)/a^4/tan(d*x+c
)^3-(a^4+6*a^2*b^2+5*b^4)/a^6/tan(d*x+c)+1/2/a^3*b/tan(d*x+c)^4+2*b*(a^2+b^
2)/a^5/tan(d*x+c)^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.31, size = 225, normalized size = 1.03

$$\frac{\frac{9\,a^4b\tan(dx+c)-60\,(a^4b+4\,a^2b^3+3\,b^5)\tan(dx+c)^5-6\,a^5-30\,(a^5+4\,a^3b^2+3\,ab^4)\tan(dx+c)^4+10\,(4\,a^4b+3\,a^2b^3)\tan(dx+c)^3-5\,(4\,a^5+3\,a^3b^2)\tan(dx+c)^2}{a^6b\tan(dx+c)^6+a^7\tan(dx+c)^5}+\frac{60\,(a^4b+4\,a^2b^3+3\,b^5)\log(b\tan(dx+c)+a)}{a^7}-\frac{60\,(a^4b+4\,a^2b^3+3\,b^5)\log(\tan(dx+c))}{a^7}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^5
 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b + 3*
a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*tan(
d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b*tan
(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/a^7)/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.
time = 0.39, size = 787, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90*
a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6 +
 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6
)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3
*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a
^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5
)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c)
)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) -
 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3
*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*
```

```
b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5
 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (4*(4*a
^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33*a^3*b^3 + 36*
a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*cos(d*x + c))*si
n(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*
cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d*cos(d*x + c)^3 +
 a^8*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.58, size = 332, normalized size = 1.52

$$-\frac{\frac{60\,(a^4b+4\,a^2b^3+3\,b^5)\log(|\tan(dx+c)|)}{a^7}-\frac{60\,(a^4b^2+4\,a^2b^4+3\,b^6)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{30\,(2\,a^4b^2\tan(dx+c)+8\,a^2b^4\tan(dx+c)+6\,b^6\tan(dx+c)+3\,a^5b+10\,a^3b^3+7\,ab^5)}{(b\tan(dx+c)+a)a^7}-\frac{137\,a^4b\tan(dx+c)^5+548\,a^2b^3\tan(dx+c)^5+411\,b^5\tan(dx+c)^5-30\,a^5\tan(dx+c)^4-180\,a^3b^2\tan(dx+c)^4-150\,ab^4\tan(dx+c)^4+60\,a^4b\tan(dx+c)^3+60\,a^2b^3\tan(dx+c)^3-20\,a^5\tan(dx+c)^2-30\,a^3b^2\tan(dx+c)^2+15\,a^4b\tan(dx+c)-6\,a^5}{a^7\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(abs(tan(d*x + c)))/a^7 - 60*(a^4*
b^2 + 4*a^2*b^4 + 3*b^6)*log(abs(b*tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*tan(d*x + c) + 8*a^2*b^4*tan(d*x + c) + 6*b^6*tan(d*x + c) + 3*a^5*b + 1
0*a^3*b^3 + 7*a*b^5)/((b*tan(d*x + c) + a)*a^7) - (137*a^4*b*tan(d*x + c)^5
 + 548*a^2*b^3*tan(d*x + c)^5 + 411*b^5*tan(d*x + c)^5 - 30*a^5*tan(d*x + c
)^4 - 180*a^3*b^2*tan(d*x + c)^4 - 150*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(
d*x + c)^3 + 60*a^2*b^3*tan(d*x + c)^3 - 20*a^5*tan(d*x + c)^2 - 30*a^3*b^2
*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 6*a^5)/(a^7*tan(d*x + c)^5))/d
```

Mupad [B]
time = 5.01, size = 237, normalized size = 1.08

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,(a^2+3\,b^2)\,(a^2+b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(2\,a^4\,b+8\,a^2\,b^3+6\,b^5)}\right)\,(a^2+3\,b^2)\,(a^2+b^2)}{a^7\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(a^4+4\,a^2\,b^2+3\,b^4)}{a^5}+\frac{\tan(c+d\,x)^2\,(4\,a^2+3\,b^2)}{6\,a^3}-\frac{3\,b\tan(c+d\,x)}{10\,a^2}+\frac{2\,b\tan(c+d\,x)^5\,(a^4+4\,a^2\,b^2+3\,b^4)}{a^6}-\frac{b\tan(c+d\,x)^3\,(4\,a^2+3\,b^2)}{3\,a^4}}{d\,\left(b\tan\left(c+d\,x\right)^6+a\tan\left(c+d\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 3*b^2)*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^4
*b + 6*b^5 + 8*a^2*b^3)))*(a^2 + 3*b^2)*(a^2 + b^2))/(a^7*d) - (1/(5*a) + (
tan(c + d*x)^4*(a^4 + 3*b^4 + 4*a^2*b^2))/a^5 + (tan(c + d*x)^2*(4*a^2 + 3*
b^2))/(6*a^3) - (3*b*tan(c + d*x))/(10*a^2) + (2*b*tan(c + d*x)^5*(a^4 + 3*
b^4 + 4*a^2*b^2))/a^6 - (b*tan(c + d*x)^3*(4*a^2 + 3*b^2))/(3*a^4))/(d*(a*t
an(c + d*x)^5 + b*tan(c + d*x)^6))
```

3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=382

$$\frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)\,x}{16\,(a^2+b^2)^6}+\frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^6\,d}$$

```
[Out] 1/16*a*(5*a^8-180*a^6*b^2+390*a^4*b^4-68*a^2*b^6-3*b^8)*x/(a^2+b^2)^6+a^4*b
*(3*a^4-22*a^2*b^2+15*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/2*
a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5/d/(a
+b*tan(d*x+c))-1/6*cos(d*x+c)^6*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a
^2+b^2)^3/d+1/24*cos(d*x+c)^4*(6*b*(9*a^4-4*a^2*b^2-b^4)+a*(13*a^4-62*a^2*b
^2-3*b^4)*tan(d*x+c))/(a^2+b^2)^4/d-1/16*a*cos(d*x+c)^2*(24*a^3*b*(3*a^2-5*
b^2)+(11*a^6-119*a^4*b^2+65*a^2*b^4+3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A]
time = 1.28, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{\cos^6(c+dx)\left(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2)\right)}{6d\left(a^2+b^2\right)^3}-\frac{a^6b}{2d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{2a^5b(a^2-3b^2)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\left(a(13a^4-62a^2b^2-3b^4)\tan(c+dx)+6b(9a^4-4a^2b^2-b^4)\right)}{24d\left(a^2+b^2\right)^4}+\frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^6}+\frac{ax(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)}{16\left(a^2+b^2\right)^6}-\frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16d\left(a^2+b^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b
^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]
)^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Co
s[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^
2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a
^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24
*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c
+ d*x]))/(16*(a^2 + b^2)^5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\text{Subst}\left(\int \frac{x^6}{(a+x)^3(b^2+x^2)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^6(a^2-3b^2}{(a^2+b^2)^3}}{}\right.}{}$$

$$= -\frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} + \frac{\cos^4(c+dx)\,(6b(9a^4}{}$$

$$= -\frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} + \frac{\cos^4(c+dx)\,(6b(9a^4}{}$$

$$= -\frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} + \frac{\cos^4(c+dx)\,(6b(9a^4}{}$$

$$= \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6\,d} - \frac{a^6b}{2\,(a^2+b^2)^4\,d(a+b\tan(c+}$$

$$= \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6\,d} - \frac{a^6b}{2\,(a^2+b^2)^4\,d(a+b\tan(c+}$$

$$= \frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)\,x}{16\,(a^2+b^2)^6} + \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(}{(a^2+b^2)^6\,d}$$

Mathematica [A]
time = 6.71, size = 683, normalized size = 1.79

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (3*a^
4*(3*a^2 - 5*b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 -
 b^4)*Cos[c + d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6
*(a^2 + b^2)^3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 2
```

```
1*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) +
(a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 -
 (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^
2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) + (3*a*(a^4 - 4*a^2*b^2
- b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*
Cos[c + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^2*b^2 -
b^4)*(ArcTan[Tan[c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^
2)^4) - (5*a*(a^2 - 3*b^2)*((2*Cos[c + d*x]^3*Sin[c + d*x])/b + 3*b^2*(ArcT
an[Tan[c + d*x]]/b^3 + (Cos[c + d*x]*Sin[c + d*x])/b^3)))/(48*(a^2 + b^2)^3
) - a^6/(2*(a^2 + b^2)^4*(a + b*Tan[c + d*x])^2) - (2*a^5*(a^2 - 3*b^2))/((
a^2 + b^2)^5*(a + b*Tan[c + d*x]))))/d
```

Maple [A]
time = 0.56, size = 466, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+b*a^4*(3*a^4-22*a^2*b^2+15*b
^4)/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5/(a+b*tan
(d*x+c))+1/(a^2+b^2)^6*(((-11/16*a^9+27/4*b^2*a^7+27/8*b^4*a^5-17/4*b^6*a^3
-3/16*b^8*a)*tan(d*x+c)^5+(-9/2*a^8*b+3*a^6*b^3+15/2*a^4*b^5)*tan(d*x+c)^4+
(-5/6*a^9+12*b^2*a^7+2*b^4*a^5-34/3*b^6*a^3-1/2*b^8*a)*tan(d*x+c)^3+(-27/4*
a^8*b+19/2*a^6*b^3+15*a^4*b^5-3/2*a^2*b^7-1/4*b^9)*tan(d*x+c)^2+(-5/16*a^9+
21/4*b^2*a^7-3/8*b^4*a^5-23/4*b^6*a^3+3/16*b^8*a)*tan(d*x+c)-11/4*a^8*b+31/
6*a^6*b^3+13/2*a^4*b^5-3/2*a^2*b^7-1/12*b^9)/(1+tan(d*x+c)^2)^3+1/16*a*(1/2
*(-48*a^7*b+352*a^5*b^3-240*a^3*b^5)*ln(1+tan(d*x+c)^2)+(5*a^8-180*a^6*b^2+
390*a^4*b^4-68*a^2*b^6-3*b^8)*arctan(tan(d*x+c)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(372) = 744$.
time = 0.60, size = 1088, normalized size = 2.85

$$\frac{\frac{3(5a^9-180a^7b^2+390a^5b^4-68a^3b^6-3ab^8)(dx+c)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} + \frac{48(3a^8b-22a^6b^3+15a^4b^5)\log(b\tan(dx+c)+a)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} - \frac{24(3a^8b-22a^6b^3+15a^4b^5)\log(\tan(dx+c)^2+1)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} - \text{[illegible]}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(b*tan(d*x + c) + a)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
- 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12) - (252
*a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*a^5*b^4
```

```
 + 65*a^3*b^6 + 3*a*b^8)*tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3 + 5*a^4
*b^5 + 3*a^2*b^7)*tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^5*b^4 + 5
29*a^3*b^6 + 24*a*b^8)*tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^3 + 65*a^4
*b^5 + 27*a^2*b^7 + 3*b^9)*tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^2 - 2171*a^
5*b^4 + 429*a^3*b^6 + 15*a*b^8)*tan(d*x + c)^3 + 2*(357*a^8*b - 987*a^6*b^3
 + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*tan(d*x + c)^2 + (15*a^9 + 93*a^7*b^2
- 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*tan(d*x + c))/(a^12 + 5*a^10*b^2 + 1
0*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^10 + (a^10*b^2 + 5*a^8*b^4 + 10*
a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*tan(d*x + c)^8 + 2*(a^11*b + 5*a^
9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^7 + (a^1
2 + 8*a^10*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 16*a^2*b^10 + 3*b^1
2)*tan(d*x + c)^6 + 6*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3
*b^9 + a*b^11)*tan(d*x + c)^5 + 3*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*tan(d*x + c)^4 + 6*(a^11*b + 5*a^9*b^
3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^3 + (3*a^12
+ 16*a^10*b^2 + 35*a^8*b^4 + 40*a^6*b^6 + 25*a^4*b^8 + 8*a^2*b^10 + b^12)*t
an(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9
 + a*b^11)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(372) = 744$.

time = 0.47, size = 932, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^11 - 8*(a^
10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*cos(d*x + c)
^8 + 20*(2*a^10*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^11)
*cos(d*x + c)^6 - 2*(49*a^10*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^7 +
33*a^2*b^9 + 6*b^11)*cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390*a^5*
b^6 - 68*a^3*b^8 - 3*a*b^10)*d*x + (9*a^10*b - 46*a^8*b^3 + 994*a^6*b^5 + 1
44*a^4*b^7 - 43*a^2*b^9 - 2*b^11 + 3*(5*a^11 - 185*a^9*b^2 + 570*a^7*b^4 -
458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c)^2 + 24*(3*a^8*b^3 -
22*a^6*b^5 + 15*a^4*b^7 + (3*a^10*b - 25*a^8*b^3 + 37*a^6*b^5 - 15*a^4*b^7)
*cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*cos(d*x + c)*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (8*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^1
0)*cos(d*x + c)^7 - 2*(13*a^11 + 55*a^9*b^2 + 90*a^7*b^4 + 70*a^5*b^6 + 25*
a^3*b^8 + 3*a*b^10)*cos(d*x + c)^5 + (33*a^11 + 49*a^9*b^2 - 54*a^7*b^4 - 1
26*a^5*b^6 - 59*a^3*b^8 - 3*a*b^10)*cos(d*x + c)^3 - (261*a^9*b^2 - 338*a^7
*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^10 + 6*(5*a^10*b - 180*a^8*b^3 + 3
90*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^1
```

```
4 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^
12 - b^14)*d*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*
b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d*cos(d*x + c)*sin(d*x + c) + (a^12
*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^10 + 6*a^2*b^12 + b^
14)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(372) = 744.
time = 0.69, size = 923, normalized size = 2.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
+ 48*(3*a^8*b^2 - 22*a^6*b^4 + 15*a^4*b^6)*log(abs(b*tan(d*x + c) + a))/(a^
12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^11 + b^1
3) - 24*(9*a^8*b^3*tan(d*x + c)^2 - 66*a^6*b^5*tan(d*x + c)^2 + 45*a^4*b^7*
tan(d*x + c)^2 + 22*a^9*b^2*tan(d*x + c) - 140*a^7*b^4*tan(d*x + c) + 78*a^
5*b^6*tan(d*x + c) + 14*a^10*b - 72*a^8*b^3 + 34*a^6*b^5)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x +
 c) + a)^2) + (132*a^8*b*tan(d*x + c)^6 - 968*a^6*b^3*tan(d*x + c)^6 + 660*
a^4*b^5*tan(d*x + c)^6 - 33*a^9*tan(d*x + c)^5 + 324*a^7*b^2*tan(d*x + c)^5
 + 162*a^5*b^4*tan(d*x + c)^5 - 204*a^3*b^6*tan(d*x + c)^5 - 9*a*b^8*tan(d*
x + c)^5 + 180*a^8*b*tan(d*x + c)^4 - 2760*a^6*b^3*tan(d*x + c)^4 + 2340*a^
4*b^5*tan(d*x + c)^4 - 40*a^9*tan(d*x + c)^3 + 576*a^7*b^2*tan(d*x + c)^3 +
 96*a^5*b^4*tan(d*x + c)^3 - 544*a^3*b^6*tan(d*x + c)^3 - 24*a*b^8*tan(d*x
+ c)^3 + 72*a^8*b*tan(d*x + c)^2 - 2448*a^6*b^3*tan(d*x + c)^2 + 2700*a^4*b
^5*tan(d*x + c)^2 - 72*a^2*b^7*tan(d*x + c)^2 - 12*b^9*tan(d*x + c)^2 - 15*
a^9*tan(d*x + c) + 252*a^7*b^2*tan(d*x + c) - 18*a^5*b^4*tan(d*x + c) - 276
*a^3*b^6*tan(d*x + c) + 9*a*b^8*tan(d*x + c) - 720*a^6*b^3 + 972*a^4*b^5 -
```

```
72*a^2*b^7 - 4*b^9)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12)*(tan(d*x + c)^2 + 1)^3))/d
```

Mupad [B]
time = 5.84, size = 1068, normalized size = 2.80

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{34\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{99\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{108\,b^7}{{\left(a^2+b^2\right)}^5}+\frac{40\,b^9}{{\left(a^2+b^2\right)}^6}\right)}{d}-\frac{\frac{{\tan(c+d\,x)}^6\,\left(31\,a^8\,b-127\,a^6\,b^3+5\,a^4\,b^5+3\,a^2\,b^7\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^7\,\left(43\,a^7\,b^2-215\,a^5\,b^4+65\,a^3\,b^6+3\,a\,b^8\right)}{16\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^5\,\left(33\,a^9+403\,a^7\,b^2-2005\,a^5\,b^4+529\,a^3\,b^6+24\,a\,b^8\right)}{48\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^4\,\left(164\,a^8\,b-515\,a^6\,b^3+65\,a^4\,b^5+27\,a^2\,b^7+3\,b^9\right)}{12\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{a^2\,\left(63\,a^6\,b-161\,a^4\,b^3+17\,a^2\,b^5+b^7\right)}{12\,\left(a^2+b^2\right)\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{{\tan(c+d\,x)}^3\,\left(40\,a^9+335\,a^7\,b^2-2171\,a^5\,b^4+429\,a^3\,b^6+15\,a\,b^8\right)}{48\,\left(a^2+b^2\right)\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{{\tan(c+d\,x)}^2\,\left(357\,a^8\,b-987\,a^6\,b^3+125\,a^4\,b^5+31\,a^2\,b^7+2\,b^9\right)}{24\,\left(a^2+b^2\right)\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{a\tan(c+d\,x)\,\left(15\,a^8+93\,a^6\,b^2-763\,a^4\,b^4+127\,a^2\,b^6+8\,b^8\right)}{48\,\left(a^2+b^2\right)\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(3\,a^2+b^2\right)+{\tan\left(c+d\,x\right)}^6\,\left(a^2+3\,b^2\right)+a^2+{\tan\left(c+d\,x\right)}^4\,\left(3\,a^2+3\,b^2\right)+b^2\,{\tan\left(c+d\,x\right)}^8+2\,a\,b\tan\left(c+d\,x\right)+6\,a\,b\,{\tan\left(c+d\,x\right)}^3+6\,a\,b\,{\tan\left(c+d\,x\right)}^5+2\,a\,b\,{\tan\left(c+d\,x\right)}^7\right)}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}-18\,a^2\,b+a\,b^2\,3{}\mathrm{i}\right)}{32\,d\,\left(-a^6+a^5\,b\,6{}\mathrm{i}+15\,a^4\,b^2-a^3\,b^3\,20{}\mathrm{i}-15\,a^2\,b^4+a\,b^5\,6{}\mathrm{i}+b^6\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(5\,a^3-a^2\,b\,18{}\mathrm{i}+3\,a\,b^2\right)}{32\,d\,\left(-a^6\,1{}\mathrm{i}+6\,a^5\,b+a^4\,b^2\,15{}\mathrm{i}-20\,a^3\,b^3-a^2\,b^4\,15{}\mathrm{i}+6\,a\,b^5+b^6\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (34*b^3)/(a^2 + b^2)^3 + (9
9*b^5)/(a^2 + b^2)^4 - (108*b^7)/(a^2 + b^2)^5 + (40*b^9)/(a^2 + b^2)^6))/d
 - ((tan(c + d*x)^6*(31*a^8*b + 3*a^2*b^7 + 5*a^4*b^5 - 127*a^6*b^3))/(8*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^7*(3*a*b^8 + 65*a^3*b^6 - 215*a^5*b^4 + 43*a^7*b^2))/(16*(a^10 + b^10 +
 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^5*(24*a*
b^8 + 33*a^9 + 529*a^3*b^6 - 2005*a^5*b^4 + 403*a^7*b^2))/(48*(a^10 + b^10
+ 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^4*(164*
a^8*b + 3*b^9 + 27*a^2*b^7 + 65*a^4*b^5 - 515*a^6*b^3))/(12*(a^10 + b^10 +
5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a^2*(63*a^6*b + b^7 +
17*a^2*b^5 - 161*a^4*b^3))/(12*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^8 + 40*a^9 + 429*a^3*b^6 - 2171*
a^5*b^4 + 335*a^7*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^2*(357*a^8*b + 2*b^9 + 31*a^2*b^7 + 125*a^4*b
^5 - 987*a^6*b^3))/(24*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) + (a*tan(c + d*x)*(15*a^8 + 8*b^8 + 127*a^2*b^6 - 763*a^4*b^4 + 93
*a^6*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2))
)/(d*(tan(c + d*x)^2*(3*a^2 + b^2) + tan(c + d*x)^6*(a^2 + 3*b^2) + a^2 + t
an(c + d*x)^4*(3*a^2 + 3*b^2) + b^2*tan(c + d*x)^8 + 2*a*b*tan(c + d*x) + 6
*a*b*tan(c + d*x)^3 + 6*a*b*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^7)) - (log(
tan(c + d*x) + 1i)*(a*b^2*3i - 18*a^2*b + a^3*5i))/(32*d*(a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)) - (log(tan(c + d*x
) - 1i)*(3*a*b^2 - a^2*b*18i + 5*a^3))/(32*d*(6*a*b^5 + 6*a^5*b - a^6*1i +
b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i))
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=285

$$\frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{8\,(a^2+b^2)^5}+\frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}-\frac{}{2\,(a^2+b^2)^3\,d(}$$

```
[Out] 3/8*a*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+3*a^2*b*(a^4-5*a^2*b^
2+2*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/2*a^4*b/(a^2+b^2)^3/
d/(a+b*tan(d*x+c))^2-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+1/4
*cos(d*x+c)^4*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d-1/8*a*
cos(d*x+c)^2*(24*a*b*(a^2-b^2)+(5*a^4-34*a^2*b^2+9*b^4)*tan(d*x+c))/(a^2+b^
2)^4/d
```

Rubi [A]
time = 0.75, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\frac{\cos^4(c+dx)\,(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2))}{4d\,(a^2+b^2)^3}-\frac{a^4b}{2d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2}-\frac{a\cos^2(c+dx)\,(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx))}{8d\,(a^2+b^2)^4}+\frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5}-\frac{2a^3b(a^2-2b^2)}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}+\frac{3ax(a^6-25a^4b^2+35a^2b^4-3b^6)}{8\,(a^2+b^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*
b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a
^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3
*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c
 + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))
/(8*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^3(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3} - \frac{a^3}{}}{}\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2\right.}{}$$

$$= \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx}$$

$$= \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx}$$

$$= \frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{8\left(a^2+b^2\right)^5} + \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} +$$

Mathematica [A]
time = 6.51, size = 501, normalized size = 1.76

$$\frac{b\left(-\frac{a^3(a^2-5b^2)\text{ArcTan}(\tan(c+dx))}{b(a^2+b^2)^4} - \frac{3a^2(a-b)(a+b)\cos^2(c+dx)}{(a^2+b^2)^4} + \frac{(3a^2-b^2)\cos^4(c+dx)}{4(a^2+b^2)^3} - \frac{a^2\left(3a^4-15a^2b^2+6b^4-\frac{a^5-13a^3b^2+10ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2(a^2+b^2)^5} + \frac{3a^2(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^5} - \frac{a^2\left(3a^4-15a^2b^2+6b^4+\frac{a^5-13a^3b^2+10ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)}{2(a^2+b^2)^5} - \frac{a^3(a^2-5b^2)\cos(c+dx)\sin(c+dx)}{b(a^2+b^2)^4} + \frac{a(a^2-3b^2)\cos^3(c+dx)\sin(c+dx)}{4b(a^2+b^2)^3} + \frac{3a(a^2-3b^2)\left(\frac{\text{ArcTan}(\tan(c+dx))}{b}+\frac{\cos(c+dx)\sin(c+dx)}{b}\right)}{8(a^2+b^2)^3} - \frac{a^4}{2(a^2+b^2)^3(a+b\tan(c+dx))^2} - \frac{2a^3(a^2-2b^2)}{(a^2+b^2)^4(a+b\tan(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)^4)) - (3*a^2*
(a - b)*(a + b)*Cos[c + d*x]^2)/(a^2 + b^2)^4 + ((3*a^2 - b^2)*Cos[c + d*x]
^4)/(4*(a^2 + b^2)^3) - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 - (a^5 - 13*a^3*b^
2 + 10*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^
5) + (3*a^2*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^
5 - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 + (a^5 - 13*a^3*b^2 + 10*a*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - (a^3*(a^2 - 5*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 + b^2)^4) + (a*(a^2 - 3*b^2)*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) + (3*a*(a^2 - 3*b^2)*(ArcTan[Tan[
```

```
c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^2)^3) - a^4/(2*(a
^2 + b^2)^3*(a + b*Tan[c + d*x])^2) - (2*a^3*(a^2 - 2*b^2))/((a^2 + b^2)^4*
(a + b*Tan[c + d*x]))))/d
```

Maple [A]
time = 0.53, size = 331, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{b\,a^4}{2\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}+\frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}-\frac{2a^3b\left(a^2-2b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}+\frac{\left(-\frac{5}{8}a^7+\frac{29}{8}a^5b^2+\frac{25}{8}a^3\right.}{}}{}$
default	$\frac{-\frac{b\,a^4}{2\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}+\frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}-\frac{2a^3b\left(a^2-2b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}+\frac{\left(-\frac{5}{8}a^7+\frac{29}{8}a^5b^2+\frac{25}{8}a^3\right.}{}}{}$
risch	$\frac{30ia^4b^3x}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{3a^2x}{8(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{6ia^6bx}{a^{10}+5a^8b^2+10a^6b^4+10a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b*a^4/(a^2+b^2)^3/(a+b*tan(d*x+c))^2+3*a^2*b*(a^4-5*a^2*b^2+2*b^4
)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/(a+b*tan(d
*x+c))+1/(a^2+b^2)^5*(((-5/8*a^7+29/8*a^5*b^2+25/8*a^3*b^4-9/8*a*b^6)*tan(d
*x+c)^3+(-3*a^6*b+3*a^2*b^5)*tan(d*x+c)^2+(-3/8*a^7+27/8*a^5*b^2+15/8*a^3*b
^4-15/8*a*b^6)*tan(d*x+c)-9/4*a^6*b+5/4*a^4*b^3+13/4*b^5*a^2-1/4*b^7)/(1+ta
n(d*x+c)^2)^2+3/8*a*(1/2*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1+tan(d*x+c)^2)+
(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(277) = 554.
time = 0.55, size = 744, normalized size = 2.61

$$\frac{\frac{3\left(a^7-25a^5b^2+35a^3b^4-3ab^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(a^6b-5a^4b^3+2a^2b^5\right)\log(b\tan(dx+c)+a)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(a^6b-5a^4b^3+2a^2b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{38a^6b-56a^4b^3+2a^2b^5+3\left(7a^5b^2-22a^3b^4+3ab^6\right)\tan(dx+c)^5+6\left(5a^6b-12a^4b^3-a^2b^5\right)\tan(dx+c)^4+\left(5a^7+49a^5b^2-133a^3b^4+15ab^6\right)\tan(dx+c)^3+2\left(35a^6b-61a^4b^3+a^2b^5+b^7\right)\tan(dx+c)^2+\left(3a^7+22a^5b^2-73a^3b^4+4ab^6\right)\tan(dx+c)}{a^{10}+4a^8b^2+6a^6b^4+4a^4b^6+a^2b^8+\left(a^8b^2+4a^6b^4+6a^4b^6+4a^2b^8+b^{10}\right)\tan(dx+c)^6+2\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)^5+\left(a^{10}+6a^8b^2+14a^6b^4+16a^4b^6+9a^2b^8+2b^{10}\right)\tan(dx+c)^4+4\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)^3+\left(2a^{10}+9a^8b^2+16a^6b^4+14a^4b^6+6a^2b^8+b^{10}\right)\tan(dx+c)^2+2\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*log(tan(d*x + c
)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) -
(38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*b^6)*t
```

```
an(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*tan(d*x + c)^4 + (5*a^7
+ 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*tan(d*x + c)^3 + 2*(35*a^6*b - 61*a^
4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a^3*b^4 +
4*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
+ (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(
a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 +
 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 +
 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*
a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)
^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(277) = 554$.
time = 0.41, size = 705, normalized size = 2.47

$$\frac{119 a^6 b^3 - 159 a^4 b^5 - 51 a^2 b^7 + 3 b^9 + 8 (a^8 b + 4 a^6 b^3 + 6 a^4 b^5 + 4 a^2 b^7 + b^9) \cos(dx+c)^6 - 8 (5 a^8 b + 16 a^6 b^3 + 18 a^4 b^5 + 8 a^2 b^7 + b^9) \cos(dx+c)^4 + 12 (a^7 b^2 - 25 a^5 b^4 + 35 a^3 b^6 - 3 a b^8) dx - (a^8 b + 110 a^6 b^3 - 420 a^4 b^5 - 78 a^2 b^7 + 3 b^9 - 12 (a^9 - 26 a^7 b^2 + 60 a^5 b^4 - 38 a^3 b^6 + 3 a b^8) dx) \cos(dx+c)^2 + 48 (a^6 b^3 - 5 a^4 b^5 + 2 a^2 b^7 + (a^8 b - 6 a^6 b^3 + 7 a^4 b^5 - 2 a^2 b^7) \cos(dx+c)^2 + 2 (a^7 b^2 - 5 a^5 b^4 + 2 a^3 b^6) \cos(dx+c) \sin(dx+c)) \log(2 a b \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2) + 2 (4 (a^9 + 4 a^7 b^2 + 6 a^5 b^4 + 4 a^3 b^6 + a b^8) \cos(dx+c)^5 - 2 (5 a^9 + 12 a^7 b^2 + 6 a^5 b^4 - 4 a^3 b^6 - 3 a b^8) \cos(dx+c)^3 + (77 a^7 b^2 - 69 a^5 b^4 + 63 a^3 b^6 - 15 a b^8 + 12 (a^8 b - 25 a^6 b^3 + 35 a^4 b^5 - 3 a^2 b^7) dx) \cos(dx+c)) \sin(dx+c)}{32 ((a^{12} + 4 a^{10} b^2 + 5 a^8 b^4 - 5 a^4 b^8 - 4 a^2 b^{10} - b^{12}) d \cos(dx+c)^2 + 2 (a^{11} b + 5 a^9 b^3 + 10 a^7 b^5 + 10 a^5 b^7 + 5 a^3 b^9 + a b^{11}) d \cos(dx+c) \sin(dx+c) + (a^{10} b^2 + 5 a^8 b^4 + 10 a^6 b^6 + 10 a^4 b^8 + 5 a^2 b^{10} + b^{12}) d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3 +
18*a^4*b^5 + 8*a^2*b^7 + b^9)*cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4 + 3
5*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*b^7
+ 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*x)*co
s(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b^3 + 7
*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^3*b^6)*
cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a
*b^8)*cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 - 3*a*
b^8)*cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15*a*b^8 + 12
*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*cos(d*x + c))*sin(d*x +
 c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos
(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 +
 a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 +
 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(277) = 554.
time = 0.67, size = 588, normalized size = 2.06

$$\frac{\frac{3\,(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6)\,(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{12\,(a^6b-5\,a^4b^3+2\,a^2b^5)\,\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{24\,(a^6b^2-5\,a^4b^4+2\,a^2b^6)\,\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}-\frac{21\,a^5b^2\tan(dx+c)^5-66\,a^3b^4\tan(dx+c)^5+9\,ab^6\tan(dx+c)^5+30\,a^6b\tan(dx+c)^4-72\,a^4b^3\tan(dx+c)^4-6\,a^2b^5\tan(dx+c)^4+5\,a^7\tan(dx+c)^3+49\,a^5b^2\tan(dx+c)^3-133\,a^3b^4\tan(dx+c)^3+15\,ab^6\tan(dx+c)^3+70\,a^6b\tan(dx+c)^2-122\,a^4b^3\tan(dx+c)^2+2\,a^2b^5\tan(dx+c)^2+2\,b^7\tan(dx+c)^2+3\,a^7\tan(dx+c)+22\,a^5b^2\tan(dx+c)-73\,a^3b^4\tan(dx+c)+4\,ab^6\tan(dx+c)+38\,a^6b-56\,a^4b^3+2\,a^2b^5}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*log(abs(b*tan
(d*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 +
 b^11) - (21*a^5*b^2*tan(d*x + c)^5 - 66*a^3*b^4*tan(d*x + c)^5 + 9*a*b^6*t
an(d*x + c)^5 + 30*a^6*b*tan(d*x + c)^4 - 72*a^4*b^3*tan(d*x + c)^4 - 6*a^2
*b^5*tan(d*x + c)^4 + 5*a^7*tan(d*x + c)^3 + 49*a^5*b^2*tan(d*x + c)^3 - 13
3*a^3*b^4*tan(d*x + c)^3 + 15*a*b^6*tan(d*x + c)^3 + 70*a^6*b*tan(d*x + c)^
2 - 122*a^4*b^3*tan(d*x + c)^2 + 2*a^2*b^5*tan(d*x + c)^2 + 2*b^7*tan(d*x +
 c)^2 + 3*a^7*tan(d*x + c) + 22*a^5*b^2*tan(d*x + c) - 73*a^3*b^4*tan(d*x +
 c) + 4*a*b^6*tan(d*x + c) + 38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2
+ b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 5.34, size = 717, normalized size = 2.52

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\left(\frac{3\,b}{(a^2+b^2)^2}-\frac{24\,b^3}{(a^2+b^2)^3}+\frac{45\,b^5}{(a^2+b^2)^4}-\frac{24\,b^7}{(a^2+b^2)^5}\right)}{d}-\frac{\frac{19\,a^6\,b-28\,a^4\,b^3+a^2\,b^5}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^2\,(35\,a^6\,b-61\,a^4\,b^3+a^2\,b^5+b^7)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}-\frac{3\tan(c+d\,x)^4\,(-5\,a^6\,b+12\,a^4\,b^3+a^2\,b^5)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{3\tan(c+d\,x)^5\,(7\,a^5\,b^2-22\,a^3\,b^4+3\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^3\,(5\,a^7+49\,a^5\,b^2-133\,a^3\,b^4+15\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{a\tan(c+d\,x)\,(3\,a^6+22\,a^4\,b^2-73\,a^2\,b^4+4\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}}{d\,\left(\tan(c+d\,x)^2\,(2\,a^2+b^2)+\tan(c+d\,x)^4\,(a^2+2\,b^2)+a^2+b^2\tan(c+d\,x)^6+2\,a\,b\tan(c+d\,x)+4\,a\,b\tan(c+d\,x)^3+2\,a\,b\tan(c+d\,x)^5\right)}-\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(a^2\,1\mathrm{i}+3\,b\,a)}{16\,d\,(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i})}-\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(3\,a\,b-a^2\,1\mathrm{i})}{16\,d\,(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (24*b^3)/(a^2 + b^2)^3 + (4
5*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5))/d - ((19*a^6*b + a^2*b^5 -
28*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c +
d*x)^2*(35*a^6*b + b^7 + a^2*b^5 - 61*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 +
 6*a^4*b^4 + 4*a^6*b^2)) - (3*tan(c + d*x)^4*(a^2*b^5 - 5*a^6*b + 12*a^4*b^
3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d*x)^5
*(3*a*b^6 - 22*a^3*b^4 + 7*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^6 + 5*a^7 - 133*a^3*b^4 + 49*a^5*b^
2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a*tan(c + d*x)*(
3*a^6 + 4*b^6 - 73*a^2*b^4 + 22*a^4*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4
*b^4 + 4*a^6*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2
```

```
+ 2*b^2) + a^2 + b^2*tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*
x)^3 + 2*a*b*tan(c + d*x)^5)) - (3*log(tan(c + d*x) - 1i)*(3*a*b + a^2*1i))
/(16*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) - (3
*log(tan(c + d*x) + 1i)*(3*a*b - a^2*1i))/(16*d*(5*a*b^4 - a^4*b*5i + a^5 -
 b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=206

$$\frac{a(a^4-14a^2b^2+9b^4)\,x}{2\,(a^2+b^2)^4}+\frac{b(3a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{a^2b}{2\,(a^2+b^2)^2\,d(a+b\tan(c+$$

```
[Out] 1/2*a*(a^4-14*a^2*b^2+9*b^4)*x/(a^2+b^2)^4+b*(3*a^4-8*a^2*b^2+b^4)*ln(a*cos
(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c)
)^2-2*a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(b*(3*a
^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A]
time = 0.32, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{a^2b}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}-\frac{2ab(a^2-b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\,(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2))}{2d\,(a^2+b^2)^3}+\frac{b(3a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{ax(a^4-14a^2b^2+9b^4)}{2\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2
*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)
^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*
b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^3(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}}{}\right.}{}$$

$$= -\frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a-}\right.\right.}{}$$

$$= \frac{b(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}$$

$$= \frac{b(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}$$

$$= \frac{a(a^4-14a^2b^2+9b^4)\,x}{2\left(a^2+b^2\right)^4} + \frac{b(3a^4-8a^2b^2+b^4)\log(\cos(c+dx))}{(a^2+b^2)^4 d} + \frac{b(3a^4-8a^2b^2}{}$$

Mathematica [A]

time = 4.22, size = 316, normalized size = 1.53

$$-\frac{b\left(\frac{a(a^2-3b^2)(a^2+b^2)\text{ArcTan}(\tan(c+dx))}{b}+(3a^2-b^2)(a^2+b^2)\cos^2(c+dx)+\left(3a^4-8a^2b^2+b^4-\frac{a^5-8a^3b^2+3ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))+\left(3a^4-8a^2b^2+b^4+\frac{a^5-8a^3b^2+3ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+\frac{a(a^2-3b^2)(a^2+b^2)\sin(2(c+dx))}{2b}+\frac{a^2(a^2+b^2)^2}{(a+b\tan(c+dx))^2}+\frac{4(a^5-ab^4)}{a+b\tan(c+dx)}\right)}{2(a^2+b^2)^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b^
2)*(a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b^2
 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*a^2
*b^2 + b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 - 8*a
^3*b^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (a*(a^2 -
3*b^2)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (a^2*(a^2 + b^2)^2)/(a + b*Tan
[c + d*x])^2 + (4*(a^5 - a*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^4*d)
```

Maple [A]
time = 0.46, size = 236, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{a^2 b}{2\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}+\frac{b\left(3a^4-8a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{2ab\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}+\frac{\left(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4\right)\tan(}{1+\tan^2(}}{d}$
default	$\frac{-\frac{a^2 b}{2\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}+\frac{b\left(3a^4-8a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{2ab\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}+\frac{\left(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4\right)\tan(}{1+\tan^2(}}{d}$
risch	$-\frac{ixb}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{xa}{2(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-3ib\,a^2+ib^3+a^3-3b^2a)d}-\frac{i\mathrm{e}^{-2i(d}}{8(3ib\,a^2-ib^3+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+b*(3*a^4-8*a^2*b^2+b^4)/(a^2
+b^2)^4*ln(a+b*tan(d*x+c))-2*a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))+1/(
a^2+b^2)^4*(((-1/2*a^5+a^3*b^2+3/2*a*b^4)*tan(d*x+c)-3/2*a^4*b-a^2*b^3+1/2*
b^5)/(1+tan(d*x+c)^2)+1/4*(-6*a^4*b+16*a^2*b^3-2*b^5)*ln(1+tan(d*x+c)^2)+1/
2*(a^5-14*a^3*b^2+9*a*b^4)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(200) = 400.
time = 0.54, size = 463, normalized size = 2.25

$$\frac{\frac{(a^5-14a^3b^2+9ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{2(3a^4b-8a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{(3a^4b-8a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8a^4b-4a^2b^3+(5a^3b^2-7ab^4)\tan(dx+c)^3+(7a^4b-6a^2b^3-b^5)\tan(dx+c)^2+(a^5+7a^3b^2-6ab^4)\tan(dx+c)}{a^8+3a^6b^2+3a^4b^4+a^2b^6+(a^6b^2+3a^4b^4+3a^2b^6+b^8)\tan(dx+c)^4+2(a^7b+3a^5b^3+3a^3b^5+ab^7)\tan(dx+c)^3+(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)\tan(dx+c)^2+2(a^7b+3a^5b^3+3a^3b^5+ab^7)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) + 2*(3*a^4*b - 8*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)
*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) -
(8*a^4*b - 4*a^2*b^3 + (5*a^3*b^2 - 7*a*b^4)*tan(d*x + c)^3 + (7*a^4*b - 6*
a^2*b^3 - b^5)*tan(d*x + c)^2 + (a^5 + 7*a^3*b^2 - 6*a*b^4)*tan(d*x + c))/(
a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c
)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a
^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(200) = 400$.
time = 0.38, size = 526, normalized size = 2.55

$$\frac{13\,a^4b^3 - 8\,a^2b^5 - b^7 - 2\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos(dx+c)^4 + 2\,(a^5b^2 - 14\,a^3b^4 + 9\,ab^6)dx - (a^6b + 23\,a^4b^3 - 21\,a^2b^5 - 3\,b^7 - 2\,(a^7 - 15\,a^5b^2 + 23\,a^3b^4 - 9\,ab^6)dx)\cos(dx+c)^2 + 2\,(3\,a^4b^3 - 8\,a^2b^5 + b^7 + (3\,a^6b - 11\,a^4b^3 + 9\,a^2b^5 - b^7)\cos(dx+c)^2 + 2\,(3\,a^5b^2 - 8\,a^3b^4 + ab^6)\cos(dx+c)\sin(dx+c))\log(2\,ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 + b^2) - 2\,((a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos(dx+c)^3 - 2\,(4\,a^5b^2 - 3\,a^3b^4 + 3\,ab^6 + (a^6b - 14\,a^4b^3 + 9\,a^2b^5)dx)\cos(dx+c))\sin(dx+c)}{4\,((a^{10} + 3\,a^8b^2 + 2\,a^6b^4 - 2\,a^4b^6 - 3\,a^2b^8 - b^{10})d\cos(dx+c)^2 + 2\,(a^9b + 4\,a^7b^3 + 6\,a^5b^5 + 4\,a^3b^7 + ab^9)d\cos(dx+c)\sin(dx+c) + (a^8b^2 + 4\,a^6b^4 + 6\,a^4b^6 + 4\,a^2b^8 + b^{10})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
*cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^4*
b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d*x)
*cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^3 +
9*a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*cos(d*x
 + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d
*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3
- 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*b^5)*d*x
)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 -
3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a
^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*
b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(200) = 400.
time = 0.68, size = 482, normalized size = 2.34

$$\frac{\frac{(a^5-14\,a^3b^2+9\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{(3\,a^4b-8\,a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{2\,(3\,a^4b^2-8\,a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}+\frac{3\,a^4b\tan(dx+c)^2-8\,a^2b^3\tan(dx+c)^2+b^5\tan(dx+c)^2-a^5\tan(dx+c)+2\,a^3b^2\tan(dx+c)+3\,ab^4\tan(dx+c)-10\,a^2b^3+2\,b^5}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\left(\tan(dx+c)^2+1\right)}-\frac{9\,a^4b^3\tan(dx+c)^2-24\,a^2b^5\tan(dx+c)^2+3\,b^7\tan(dx+c)^2+22\,a^5b^2\tan(dx+c)-48\,a^3b^4\tan(dx+c)+2\,ab^6\tan(dx+c)+14\,a^6b-22\,a^4b^3}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*log(tan(d*x + c)^2 + 1)/(a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 + b^
6)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9) + (3*a^4*b*tan(d*x + c)^2 - 8*a^2*b^3*tan(d*x + c)^2 + b^5*tan(d*x +
 c)^2 - a^5*tan(d*x + c) + 2*a^3*b^2*tan(d*x + c) + 3*a*b^4*tan(d*x + c) -
10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d
*x + c)^2 + 1)) - (9*a^4*b^3*tan(d*x + c)^2 - 24*a^2*b^5*tan(d*x + c)^2 + 3
*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) - 48*a^3*b^4*tan(d*x + c) + 2
*a*b^6*tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 4.62, size = 433, normalized size = 2.10

$$\frac{\frac{\tan(c+d\,x)^2\,\left(-7\,a^4\,b+6\,a^2\,b^3+b^5\right)}{2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)^3\,\left(7\,a\,b^4-5\,a^3\,b^2\right)}{2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{2\,a^2\,\left(2\,a^2\,b-b^3\right)}{(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}-\frac{a\tan(c+d\,x)\,\left(a^4+7\,a^2\,b^2-6\,b^4\right)}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(a^2+b^2\right)+a^2+b^2\tan\left(c+d\,x\right)^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^3\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{(a^2+b^2)^2}-\frac{14\,b^3}{(a^2+b^2)^3}+\frac{12\,b^5}{(a^2+b^2)^4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-2\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a-b\,2\mathrm{i}\right)}{4\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c + d*x)^2*(b^5 - 7*a^4*b + 6*a^2*b^3))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^3*(7*a*b^4 - 5*a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) - (2*a^2*(2*a^2*b - b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*
b^2)) - (a*tan(c + d*x)*(a^4 - 6*b^4 + 7*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^
4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c + d*x)^4
+ 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(a + b*tan(c + d*x))*((
3*b)/(a^2 + b^2)^2 - (14*b^3)/(a^2 + b^2)^3 + (12*b^5)/(a^2 + b^2)^4))/d +
(log(tan(c + d*x) + 1i)*(a*1i - 2*b))/(4*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4
 - 6*a^2*b^2)) + (log(tan(c + d*x) - 1i)*(a - b*2i))/(4*d*(4*a*b^3 - 4*a^3*
b + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{\cot(c+dx)}{a^3d}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{2b}{a^3d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^3/d-3*b*ln(tan(d*x+c))/a^4/d+3*b*ln(a+b*tan(d*x+c))/a^4/d-1/2
*b/a^2/d/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 46}

$$-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{2b}{a^3d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^3d}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Cot[c + d*x]/(a^3*d)) - (3*b*Log[Tan[c + d*x]])/(a^4*d) + (3*b*Log[a + b*
Tan[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*Tan[c + d*x])^2) - (2*b)/(a^3*d*
(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3b\log(\tan(c+dx))}{a^4d} + \frac{3b\log(a+b\tan(c+dx))}{a^4d} - \frac{}{2a^2d(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(95) = 190$.
time = 2.86, size = 241, normalized size = 2.54

$$\frac{-2a^3(a^2+b^2)\cot(c+dx)+b(-2a^2(a^2+b^2)(2+3\log(\sin(c+dx))-3\log(a\cos(c+dx)+b\sin(c+dx)))-a^2b^2\sec^2(c+dx)+2ab(2a^2+b^2-6(a^2+b^2)\log(\sin(c+dx))+6(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)-2b^2(-3a^2-2b^2+3(a^2+b^2)\log(\sin(c+dx))-3(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan^2(c+dx))}{2a^4(a^2+b^2)d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 + b^2)*Cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*Log[Sin[c +
 d*x]] - 3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) - a^2*b^2*Sec[c + d*x]^2 +
 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 + b^2)*Log[a
*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 + 3*(
a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])*Tan[c + d*x]^2))/(2*a^4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.38, size = 85, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{b}{2a^2(a+b\tan(dx+c))^2}+\frac{3b\ln(a+b\tan(dx+c))}{a^4}-\frac{2b}{a^3(a+b\tan(dx+c))}-\frac{1}{a^3\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^4}}{d}$
default	$\frac{-\frac{b}{2a^2(a+b\tan(dx+c))^2}+\frac{3b\ln(a+b\tan(dx+c))}{a^4}-\frac{2b}{a^3(a+b\tan(dx+c))}-\frac{1}{a^3\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^4}}{d}$
risch	$-\frac{2i(a^4e^{4i(dx+c)}-9a^2b^2e^{4i(dx+c)}+3b^4e^{4i(dx+c)}-4ia^3b\,e^{4i(dx+c)}+9ia\,b^3e^{4i(dx+c)}+2a^4e^{2i(dx+c)}-6b^4e^{2i(dx+c)}-4ia^3b}{(e^{2i(dx+c)}-1)(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a)^2(-ib+a)^2d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b/a^2/(a+b*tan(d*x+c))^2+3/a^4*b*ln(a+b*tan(d*x+c))-2/a^3*b/(a+b*
tan(d*x+c))-1/a^3/tan(d*x+c)-3/a^4*b*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.33, size = 108, normalized size = 1.14

$$-\frac{\frac{6\,b^2\tan\left(dx+c\right)^2+9\,ab\tan\left(dx+c\right)+2\,a^2}{a^3b^2\tan\left(dx+c\right)^3+2\,a^4b\tan\left(dx+c\right)^2+a^5\tan\left(dx+c\right)}-\frac{6\,b\log\left(b\tan\left(dx+c\right)+a\right)}{a^4}+\frac{6\,b\log\left(\tan\left(dx+c\right)\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*b^2*tan(d*x + c)^2 + 9*a*b*tan(d*x + c) + 2*a^2)/(a^3*b^2*tan(d*x
+ c)^3 + 2*a^4*b*tan(d*x + c)^2 + a^5*tan(d*x + c)) - 6*b*log(b*tan(d*x + c
) + a)/a^4 + 6*b*log(tan(d*x + c))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(93) = 186.
time = 0.44, size = 565, normalized size = 5.95

$$\frac{2\left(a^7+4\,a^5b^2-2\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)^3-2\left(2\,a^5b^2-3\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)+3\left(2\left(a^5b^2+2\,a^3b^4+ab^6\right)\cos\left(dx+c\right)^3-2\left(a^5b^2+2\,a^3b^4+ab^6\right)\cos\left(dx+c\right)-\left(a^4b^3+2\,a^2b^5+b^7+\left(a^6b+a^4b^3-a^2b^5-b^7\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-3\left(2\left(a^5b^2+2\,a^3b^4+ab^6\right)\cos\left(dx+c\right)^3-2\left(a^5b^2+2\,a^3b^4+ab^6\right)\cos\left(dx+c\right)-\left(a^4b^3+2\,a^2b^5+b^7+\left(a^6b+a^4b^3-a^2b^5-b^7\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)-\left(5\,a^4b^3+3\,a^2b^5-4\left(a^6b+5\,a^4b^3+3\,a^2b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{2\left(2\left(a^9b+2\,a^7b^3+a^5b^5\right)d\cos\left(dx+c\right)^3-2\left(a^9b+2\,a^7b^3+a^5b^5\right)d\cos\left(dx+c\right)-\left(\left(a^{10}+a^8b^2-a^6b^4-a^4b^6\right)d\cos\left(dx+c\right)^2+\left(a^8b^2+2\,a^6b^4+a^4b^6\right)d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b^
2 - 3*a^3*b^4 - 3*a*b^6)*cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 +
2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
 + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^
3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3
 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1
/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cos(d*x +
c)^2)*sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^3 - 2*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) - ((a^10 + a^8*b^2 - a^6*b^4 -
a^4*b^6)*d*cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.60, size = 113, normalized size = 1.19

$$\frac{\frac{6\,b\log(|b\tan(dx+c)+a|)}{a^4}-\frac{6\,b\log(|\tan(dx+c)|)}{a^4}+\frac{2\,(3\,b\tan(dx+c)-a)}{a^4\tan(dx+c)}-\frac{9\,b^3\tan(dx+c)^2+22\,ab^2\tan(dx+c)+14\,a^2b}{(b\tan(dx+c)+a)^2a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4
+ 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 22*
a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d
```

Mupad [B]
time = 3.87, size = 99, normalized size = 1.04

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^4\,d}-\frac{\frac{1}{a}+\frac{3\,b^2\tan(c+d\,x)^2}{a^3}+\frac{9\,b\tan(c+d\,x)}{2\,a^2}}{d\,\left(a^2\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^2+b^2\tan\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (6*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^4*d) - (1/a + (3*b^2*tan(c + d*x)^
2)/a^3 + (9*b*tan(c + d*x))/(2*a^2))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d*x
)^3 + 2*a*b*tan(c + d*x)^2))
```

3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{(a^2+6b^2)\cot(c+dx)}{a^5d}+\frac{3b\cot^2(c+dx)}{2a^4d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d}+\frac{b(3a^2+10b^2)\log(}{a}$$

```
[Out] -(a^2+6*b^2)*cot(d*x+c)/a^5/d+3/2*b*cot(d*x+c)^2/a^4/d-1/3*cot(d*x+c)^3/a^3
/d-b*(3*a^2+10*b^2)*ln(tan(d*x+c))/a^6/d+b*(3*a^2+10*b^2)*ln(a+b*tan(d*x+c)
)/a^6/d-1/2*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/d/(a+b
*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{3b\cot^2(c+dx)}{2a^4d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d}+\frac{b(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6d}-\frac{2b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))}-\frac{(a^2+6b^2)\cot(c+dx)}{a^5d}-\frac{b(a^2+b^2)}{2a^4d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) -
Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) +
 (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*
a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d
*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^3x^4} - \frac{3b^2}{a^4x^3} + \frac{a^2+6b^2}{a^5x^2} + \frac{-3a^2-10b^2}{a^6x} + \frac{a^2+b^2}{a^4(a+x)^3} + \frac{2(a^2+2b^2)}{a^5(a+x)^2} + \frac{3a^2+10b^2}{a^6(a+x)}\right)\right.}{d} \\
&= -\frac{(a^2+6b^2)\cot(c+dx)}{a^5d} + \frac{3b\cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d} - \frac{b(3a^2+10b^2)\log}{a^6}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(178) = 356$.
time = 3.93, size = 456, normalized size = 2.56

$$
-\frac{b^3\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{2a^4d(a+b\tan(c+dx))^3} - \frac{\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^3d(a+b\tan(c+dx))^3} - \frac{2(a^2\cos(c+dx)+9b^2\cos(c+dx))\csc(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^5d(a+b\tan(c+dx))^3} + \frac{3b\csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{2a^4d(a+b\tan(c+dx))^3} + \frac{(-3a^2b-10b^3)\log(\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3} + \frac{(3a^2b+10b^3)\log(a\cos(c+dx)+b\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3} + \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2(3a^2b^2\sin(c+dx)+4b^4\sin(c+dx))}{a^6d(a+b\tan(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b^3*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a^4*d*(a + b*T
an[c + d*x])^3) - (Csc[c + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3)/(3*a^3*d*(a + b*Tan[c + d*x])^3) - (2*(a^2*Cos[c + d*x] + 9*b^2*
Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(3*a^5*d*(a + b*Tan[c + d*x])^3) + (3*b*Csc[c + d*x]^2*Sec[c + d*x]^3*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(2*a^4*d*(a + b*Tan[c + d*x])^3) + ((-3
*a^2*b - 10*b^3)*Log[Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + ((3*a^2*b + 10*b^3)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2*(3*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*x]))/(a^6*d*(a + b*
Tan[c + d*x])^3)
```

Maple [A]
time = 0.41, size = 158, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6} - \frac{\left(a^2+b^2\right)b}{2a^4(a+b\tan(dx+c))^2} - \frac{2b\left(a^2+2b^2\right)}{a^5(a+b\tan(dx+c))} - \frac{1}{3a^3\tan(dx+c)^3} - \frac{a^2+6b^2}{a^5\tan(dx+c)} + \frac{3b}{2a^4\tan(dx+c)^2}}{d}$
default	$\frac{\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6} - \frac{\left(a^2+b^2\right)b}{2a^4(a+b\tan(dx+c))^2} - \frac{2b\left(a^2+2b^2\right)}{a^5(a+b\tan(dx+c))} - \frac{1}{3a^3\tan(dx+c)^3} - \frac{a^2+6b^2}{a^5\tan(dx+c)} + \frac{3b}{2a^4\tan(dx+c)^2}}{d}$
risch	$-\frac{2i\left(15a^4b\,e^{4i(dx+c)}+2a^4b+29a^2b^3+30b^5+9a^4b\,e^{6i(dx+c)}+104a^2b^3e^{4i(dx+c)}-6a^2b^3e^{6i(dx+c)}+6ia^5e^{6i(dx+c)}+10ia^5e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(3*a^2+10*b^2)/a^6*ln(a+b*tan(d*x+c))-1/2*(a^2+b^2)*b/a^4/(a+b*tan(d
*x+c))^2-2*b*(a^2+2*b^2)/a^5/(a+b*tan(d*x+c))-1/3/a^3/tan(d*x+c)^3-(a^2+6*b
^2)/a^5/tan(d*x+c)+3/2/a^4*b/tan(d*x+c)^2-b*(3*a^2+10*b^2)/a^6*ln(tan(d*x+c
)))
```

Maxima [A]
time = 0.32, size = 192, normalized size = 1.08

$$\frac{\frac{5\,a^3b\tan(dx+c)-6\,(3\,a^2b^2+10\,b^4)\tan(dx+c)^4-2\,a^4-9\,(3\,a^3b+10\,ab^3)\tan(dx+c)^3-2\,(3\,a^4+10\,a^2b^2)\tan(dx+c)^2}{a^5b^2\tan(dx+c)^5+2\,a^6b\tan(dx+c)^4+a^7\tan(dx+c)^3}+\frac{6\,(3\,a^2b+10\,b^3)\log(b\tan(dx+c)+a)}{a^6}-\frac{6\,(3\,a^2b+10\,b^3)\log(\tan(dx+c))}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((5*a^3*b*tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*tan(d*x + c)^4 - 2*a^4
- 9*(3*a^3*b + 10*a*b^3)*tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*tan(d*x +
c)^2)/(a^5*b^2*tan(d*x + c)^5 + 2*a^6*b*tan(d*x + c)^4 + a^7*tan(d*x + c)^3
) + 6*(3*a^2*b + 10*b^3)*log(b*tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*b^3)
*log(tan(d*x + c))/a^6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(172) = 344$.
time = 0.40, size = 811, normalized size = 4.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^7 + 27*a^5*b^2 + a^3*b^4 - 30*a*b^6)*cos(d*x + c)^5 - 2*(3*a^7
+ 43*a^5*b^2 - 8*a^3*b^4 - 60*a*b^6)*cos(d*x + c)^3 + 6*(5*a^5*b^2 - 3*a^3*
b^4 - 10*a*b^6)*cos(d*x + c) + 3*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos
(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5 + 10
*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*
b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*
b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*(2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4
+ 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x
+ c) + (3*a^4*b^3 + 13*a^2*b^5 + 10*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5
 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos
```

```
(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (24*a^4*b^3 + 3
0*a^2*b^5 + 4*(2*a^6*b + 29*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c)^4 - 3*(a^6*b
 + 45*a^4*b^3 + 50*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + a^7*b
^3)*d*cos(d*x + c)^5 - 4*(a^9*b + a^7*b^3)*d*cos(d*x + c)^3 + 2*(a^9*b + a^
7*b^3)*d*cos(d*x + c) - ((a^10 - a^6*b^4)*d*cos(d*x + c)^4 - (a^10 - a^8*b^
2 - 2*a^6*b^4)*d*cos(d*x + c)^2 - (a^8*b^2 + a^6*b^4)*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.60, size = 237, normalized size = 1.33

$$-\frac{\frac{6\,(3\,a^2b+10\,b^3)\log(|\tan(dx+c)|)}{a^6}-\frac{6\,(3\,a^2b^2+10\,b^4)\log(|b\tan(dx+c)+a|)}{a^6b}+\frac{3\left(9\,a^2b^3\tan(dx+c)^2+30\,b^5\tan(dx+c)^2+22\,a^3b^2\tan(dx+c)+68\,ab^4\tan(dx+c)+14\,a^4b+39\,a^2b^3\right)}{(b\tan(dx+c)+a)^2a^6}-\frac{33\,a^2b\tan(dx+c)^3+110\,b^3\tan(dx+c)^3-6\,a^3\tan(dx+c)^2-36\,ab^2\tan(dx+c)^2+9\,a^2b\tan(dx+c)-2\,a^3}{a^6\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*(3*a^2*b + 10*b^3)*log(abs(tan(d*x + c)))/a^6 - 6*(3*a^2*b^2 + 10*b
^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b) + 3*(9*a^2*b^3*tan(d*x + c)^2 + 30
*b^5*tan(d*x + c)^2 + 22*a^3*b^2*tan(d*x + c) + 68*a*b^4*tan(d*x + c) + 14*
a^4*b + 39*a^2*b^3)/((b*tan(d*x + c) + a)^2*a^6) - (33*a^2*b*tan(d*x + c)^3
 + 110*b^3*tan(d*x + c)^3 - 6*a^3*tan(d*x + c)^2 - 36*a*b^2*tan(d*x + c)^2
+ 9*a^2*b*tan(d*x + c) - 2*a^3)/(a^6*tan(d*x + c)^3))/d
```

Mupad [B]
time = 4.37, size = 200, normalized size = 1.12

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(3\,a^2+10\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(3\,a^2\,b+10\,b^3)}\right)\,(3\,a^2+10\,b^2)}{a^6\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(3\,a^2+10\,b^2)}{3\,a^3}-\frac{5\,b\tan(c+d\,x)}{6\,a^2}+\frac{b^2\tan(c+d\,x)^4\,(3\,a^2+10\,b^2)}{a^5}+\frac{3\,b\tan(c+d\,x)^3\,(3\,a^2+10\,b^2)}{2\,a^4}}{d\,\left(a^2\tan(c+d\,x)^3+2\,a\,b\tan(c+d\,x)^4+b^2\tan(c+d\,x)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(3*a^2 + 10*b^2)*(a + 2*b*tan(c + d*x)))/(a*(3*a^2*b + 10*b^3
)))*(3*a^2 + 10*b^2))/(a^6*d) - (1/(3*a) + (tan(c + d*x)^2*(3*a^2 + 10*b^2)
)/(3*a^3) - (5*b*tan(c + d*x))/(6*a^2) + (b^2*tan(c + d*x)^4*(3*a^2 + 10*b^
2))/a^5 + (3*b*tan(c + d*x)^3*(3*a^2 + 10*b^2))/(2*a^4))/(d*(a^2*tan(c + d*
x)^3 + b^2*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^4))
```

3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=265

$$-\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d}+\frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d}-\frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d}+\frac{3b\cot^4(c+dx)}{4a^4d}-c$$

```
[Out] -(a^4+12*a^2*b^2+15*b^4)*cot(d*x+c)/a^7/d+b*(3*a^2+5*b^2)*cot(d*x+c)^2/a^6/
d-2/3*(a^2+3*b^2)*cot(d*x+c)^3/a^5/d+3/4*b*cot(d*x+c)^4/a^4/d-1/5*cot(d*x+c
)^5/a^3/d-b*(3*a^4+20*a^2*b^2+21*b^4)*ln(tan(d*x+c))/a^8/d+b*(3*a^4+20*a^2*
b^2+21*b^4)*ln(a+b*tan(d*x+c))/a^8/d-1/2*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c
))^2-2*b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{3b\cot^4(c+dx)}{4a^4d}-\frac{\cot^5(c+dx)}{5a^3d}-\frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))}-\frac{b(a^2+b^2)^2}{2a^6d(a+b\tan(c+dx))^2}+\frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d}-\frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d}-\frac{b(3a^4+20a^2b^2+21b^4)\log(\tan(c+dx))}{a^8d}+\frac{b(3a^4+20a^2b^2+21b^4)\log(a+b\tan(c+dx))}{a^8d}-\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*
Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b
*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*
b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)
*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c
 + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\text{Subst}\Big(\int \frac{(b^2+x^2)^2}{x^6(a+x)^3}\,dx, x, b\tan(c+dx)\Big)}{d}$$

$$= \frac{b\text{Subst}\Big(\int \Big(\frac{b^4}{a^3x^6} - \frac{3b^4}{a^4x^5} + \frac{2b^2(a^2+3b^2)}{a^5x^4} - \frac{2(3a^2b^2+5b^4)}{a^6x^3} + \frac{a^4+12a^2b^2+15b^4}{a^7x^2} + \frac{-3a^4-20a^2b}{a^8x}}{d}$$

$$= -\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{2(a^2+3b^2)}{3a}$$

Mathematica [A]
time = 4.99, size = 494, normalized size = 1.86

$$-\frac{\csc^5(c+dx)\left(\sec^2(c+dx)\left((8a^7+567a^5b^2+630a^3b^4-1215ab^6)\cos(3(c+dx))-(24a^7+619a^5b^2+630a^3b^4-675ab^6)\cos(5(c+dx))+8a^7\cos(7(c+dx))+187a^5b^2\cos(7(c+dx))+210a^3b^4\cos(7(c+dx))-135ab^6\cos(7(c+dx))-126a^6b\sin(3(c+dx))+1665a^4b^3\sin(3(c+dx))+4635a^2b^5\sin(3(c+dx))+1890b^7\sin(3(c+dx))+10a^6b\sin(5(c+dx))-1215a^4b^3\sin(5(c+dx))-2565a^2b^5\sin(5(c+dx))-630b^7\sin(5(c+dx))+16a^6b\sin(7(c+dx))+345a^4b^3\sin(7(c+dx))+585a^2b^5\sin(7(c+dx))+90b^7\sin(7(c+dx))\right)+960b(3a^4+20a^2b^2+21b^4)(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))\sin^5(c+dx)(a+b\tan(c+dx))^2+5\sec(c+dx)(40a^7-27a^5b^2-42a^3b^4+135ab^6-3b(8a^6+89a^4b^2+345a^2b^4+210b^6)\tan(c+dx))\right)}{960a^8d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*(Sec[c + d*x]^2*((8*a^7 + 567*a^5*b^2 + 630*a^3*b^4
- 1215*a*b^6)*Cos[3*(c + d*x)] - (24*a^7 + 619*a^5*b^2 + 630*a^3*b^4 - 675*
a*b^6)*Cos[5*(c + d*x)] + 8*a^7*Cos[7*(c + d*x)] + 187*a^5*b^2*Cos[7*(c + d
*x)] + 210*a^3*b^4*Cos[7*(c + d*x)] - 135*a*b^6*Cos[7*(c + d*x)] - 126*a^6*
b*Sin[3*(c + d*x)] + 1665*a^4*b^3*Sin[3*(c + d*x)] + 4635*a^2*b^5*Sin[3*(c
+ d*x)] + 1890*b^7*Sin[3*(c + d*x)] + 10*a^6*b*Sin[5*(c + d*x)] - 1215*a^4*
b^3*Sin[5*(c + d*x)] - 2565*a^2*b^5*Sin[5*(c + d*x)] - 630*b^7*Sin[5*(c + d
*x)] + 16*a^6*b*Sin[7*(c + d*x)] + 345*a^4*b^3*Sin[7*(c + d*x)] + 585*a^2*b
^5*Sin[7*(c + d*x)] + 90*b^7*Sin[7*(c + d*x)]) + 960*b*(3*a^4 + 20*a^2*b^2
+ 21*b^4)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Sin[c
+ d*x]^5*(a + b*Tan[c + d*x])^2 + 5*Sec[c + d*x]*(40*a^7 - 27*a^5*b^2 - 42*
a^3*b^4 + 135*a*b^6 - 3*b*(8*a^6 + 89*a^4*b^2 + 345*a^2*b^4 + 210*b^6)*Tan[
c + d*x])))/(a^8*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.45, size = 246, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{b\left(3a^4+20a^2b^2+21b^4\right)\ln(a+b\tan(dx+c))}{a^8} - \frac{\left(a^4+2a^2b^2+b^4\right)b}{2a^6(a+b\tan(dx+c))^2} - \frac{2b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))} - \frac{1}{5a^3\tan(dx+c)^5} - \frac{2a^2+6b^2}{3a^5\tan(dx+c)^3} - \frac{a^4+}{a^7}}{d}$
default	$\frac{\frac{b\left(3a^4+20a^2b^2+21b^4\right)\ln(a+b\tan(dx+c))}{a^8} - \frac{\left(a^4+2a^2b^2+b^4\right)b}{2a^6(a+b\tan(dx+c))^2} - \frac{2b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))} - \frac{1}{5a^3\tan(dx+c)^5} - \frac{2a^2+6b^2}{3a^5\tan(dx+c)^3} - \frac{a^4+}{a^7}}{d}$
risch	$-\frac{2i\left(-420ia^3b^3e^{8i(dx+c)}-100ia^5b\,e^{6i(dx+c)}-1530ia^3b^3e^{6i(dx+c)}+9ia^5b\,e^{4i(dx+c)}-315ia\,b^5e^{12i(dx+c)}+945ia\,b^5e^{10i(d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(a+b*tan(d*x+c))-1/2*(a^4+2*a^2*b^2+b^4)*b/a^6/(a+b*tan(d*x+c))^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+c))-1/5/a^3/tan(d*x+c)^5-1/3*(2*a^2+6*b^2)/a^5/tan(d*x+c)^3-(a^4+12*a^2*b^2+15*b^4)/a^7/tan(d*x+c)+3/4/a^4*b/tan(d*x+c)^4+b*(3*a^2+5*b^2)/a^6/tan(d*x+c)^2-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(tan(d*x+c)))

Maxima [A]
time = 0.35, size = 281, normalized size = 1.06

$$\frac{\frac{21\,a^5b\tan(dx+c)-60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\tan(dx+c)^6-12\,a^6-90\,(3\,a^5b+20\,a^3b^3+21\,ab^5)\tan(dx+c)^5-20\,(3\,a^6+20\,a^4b^2+21\,a^2b^4)\tan(dx+c)^4+5\,(20\,a^5b+21\,a^3b^3)\tan(dx+c)^3-2\,(20\,a^6+21\,a^4b^2)\tan(dx+c)^2}{a^7b^2\tan(dx+c)^7+2\,a^8b\tan(dx+c)^6+a^9\tan(dx+c)^5}+\frac{60\,(3\,a^4b+20\,a^2b^3+21\,b^5)\log(b\tan(dx+c)+a)}{a^8}-\frac{60\,(3\,a^4b+20\,a^2b^3+21\,b^5)\log(\tan(dx+c))}{a^8}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*((21*a^5*b*tan(d*x + c) - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*tan(d*x + c)^6 - 12*a^6 - 90*(3*a^5*b + 20*a^3*b^3 + 21*a*b^5)*tan(d*x + c)^5 - 20*(3*a^6 + 20*a^4*b^2 + 21*a^2*b^4)*tan(d*x + c)^4 + 5*(20*a^5*b + 21*a^3*b^3)*tan(d*x + c)^3 - 2*(20*a^6 + 21*a^4*b^2)*tan(d*x + c)^2)/(a^7*b^2*tan(d*x + c)^7 + 2*a^8*b*tan(d*x + c)^6 + a^9*tan(d*x + c)^5) + 60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(b*tan(d*x + c) + a)/a^8 - 60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(tan(d*x + c))/a^8)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(257) = 514$.
time = 0.41, size = 1018, normalized size = 3.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(4*(8*a^7 + 187*a^5*b^2 + 120*a^3*b^4 - 315*a*b^6)*cos(d*x + c)^7 - 4*(20*a^7 + 482*a^5*b^2 + 255*a^3*b^4 - 945*a*b^6)*cos(d*x + c)^5 + 10*(6*a^7 + 157*a^5*b^2 + 60*a^3*b^4 - 378*a*b^6)*cos(d*x + c)^3 - 30*(13*a^5*b^2 + 2*a^3*b^4 - 42*a*b^6)*cos(d*x + c) + 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^5

```
- 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a
*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5
 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20
*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a
^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^
3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^
5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) -
(285*a^4*b^3 + 630*a^2*b^5 - 8*(8*a^6*b + 195*a^4*b^3 + 315*a^2*b^5)*cos(d*
x + c)^6 + 10*(7*a^6*b + 330*a^4*b^3 + 567*a^2*b^5)*cos(d*x + c)^4 + 15*(a^
6*b - 135*a^4*b^3 - 252*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*a^9*b*d*c
os(d*x + c)^7 - 6*a^9*b*d*cos(d*x + c)^5 + 6*a^9*b*d*cos(d*x + c)^3 - 2*a^9
*b*d*cos(d*x + c) - (a^8*b^2*d + (a^10 - a^8*b^2)*d*cos(d*x + c)^6 - (2*a^1
0 - 3*a^8*b^2)*d*cos(d*x + c)^4 + (a^10 - 3*a^8*b^2)*d*cos(d*x + c)^2)*sin(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.63, size = 382, normalized size = 1.44

$$-\frac{\frac{60\,(3\,a^4b+20\,a^2b^3+21\,b^5)\log(|\tan(dx+c)|)}{a^8}-\frac{60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\log(|b\tan(dx+c)+a|)}{a^8b}+\frac{30\left(9\,a^4b^3\tan(dx+c)^2+60\,a^2b^5\tan(dx+c)^2+63\,b^7\tan(dx+c)^2+22\,a^5b^2\tan(dx+c)+136\,a^3b^4\tan(dx+c)+138\,ab^6\tan(dx+c)+14\,a^6b+78\,a^4b^3+76\,a^2b^5\right)}{(b\tan(dx+c)+a)^2a^8}-\frac{411\,a^4b\tan(dx+c)^5+2740\,a^2b^3\tan(dx+c)^5+2877\,b^5\tan(dx+c)^5-60\,a^5\tan(dx+c)^4-720\,a^3b^2\tan(dx+c)^4-900\,ab^4\tan(dx+c)^4+180\,a^4b\tan(dx+c)^3+300\,a^2b^3\tan(dx+c)^3-40\,a^5\tan(dx+c)^2-120\,a^3b^2\tan(dx+c)^2+45\,a^4b\tan(dx+c)-12\,a^5}{a^8\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] -1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*(
3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 30*
(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x + c)
^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan(d*x
 + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^8) -
(411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*tan(d*x
+ c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*b^4*tan
(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3 - 40*a^
5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x + c) - 12*
a^5)/(a^8*tan(d*x + c)^5))/d
```

Mupad [B]
time = 5.32, size = 297, normalized size = 1.12

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a+2\,b\tan(c+d\,x))\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a\,(3\,a^4\,b+20\,a^2\,b^3+21\,b^5)}\right)\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^8\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{3\,a^5}+\frac{\tan(c+d\,x)^2\,(20\,a^2+21\,b^2)}{30\,a^3}-\frac{7\,b\tan(c+d\,x)}{20\,a^2}+\frac{b^2\tan(c+d\,x)^6\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^7}+\frac{3\,b\tan(c+d\,x)^5\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{2\,a^6}-\frac{b\tan(c+d\,x)^3\,(20\,a^2+21\,b^2)}{12\,a^4}}{d\,\left(a^2\tan\left(c+d\,x\right)^5+2\,a\,b\tan\left(c+d\,x\right)^6+b^2\tan\left(c+d\,x\right)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(a + 2*b*tan(c + d*x))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a*(3*a
^4*b + 21*b^5 + 20*a^2*b^3)))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a^8*d) - (1/(
5*a) + (tan(c + d*x)^4*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(3*a^5) + (tan(c + d*
x)^2*(20*a^2 + 21*b^2))/(30*a^3) - (7*b*tan(c + d*x))/(20*a^2) + (b^2*tan(c
 + d*x)^6*(3*a^4 + 21*b^4 + 20*a^2*b^2))/a^7 + (3*b*tan(c + d*x)^5*(3*a^4 +
 21*b^4 + 20*a^2*b^2))/(2*a^6) - (b*tan(c + d*x)^3*(20*a^2 + 21*b^2))/(12*a
^4))/(d*(a^2*tan(c + d*x)^5 + b^2*tan(c + d*x)^7 + 2*a*b*tan(c + d*x)^6))
```

3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=366

$$\frac{(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)\,x}{8\,(a^2+b^2)^6}+\frac{4ab(a^2-b^2)\,(a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+ dx))}{(a^2+b^2)^6\,d}$$

```
[Out] 1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*a^2*b^6+3*b^8)*x/(a^2+b^2)^6+4*a*b*(
a^2-b^2)*(a^4-8*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/
3*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^3-a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a
+b*tan(d*x+c))^2-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b
^2)^4/d-1/8*cos(d*x+c)^2*(16*a*b*(2*a^4-5*a^2*b^2+b^4)+(5*a^6-65*a^4*b^2+55
*a^2*b^4-3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A]
time = 1.26, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{a^4b}{3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^3}-\frac{3a^2b(a^4-5a^2b^2+2b^4)}{d\,(a^2+b^2)^5\,(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{4d\,(a^2+b^2)^4}+\frac{4ab(a^2-b^2)\,(a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^6}-\frac{a^3b(a^2-2b^2)}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\,(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx))}{8d\,(a^2+b^2)^5}+\frac{x(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)}{8\,(a^2+b^2)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2
)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c +
d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) -
 (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x]))
 + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x
]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4)
 + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^
5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^4(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \ldots$$

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\,(1\ldots}{\ldots}$$

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\,(1\ldots}{\ldots}$$

$$= \frac{4ab(a^2-b^2)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan\ldots}$$

$$= \frac{4ab(a^2-b^2)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan\ldots}$$

$$= \frac{\left(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8\right)x}{8\left(a^2+b^2\right)^6} + \frac{4ab(a^2-b^2)\left(a^4-8a^2b^2+b^4\right)\ldots}{\left(a^2+b^2\right)^6 d}$$

Mathematica [A]
time = 6.00, size = 564, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(b*((24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x
]])/b + 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a
 - b)*(a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*
a^2*b^4 - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*L
og[Sqrt[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b
^4)*Log[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6
 + (a^7 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] +
b*Tan[c + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3*S
in[c + d*x])/b + (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c +
```

```
d*x)])/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*(2*ArcTan[Tan[c + d*x]]
 + Sin[2*(c + d*x)]))/(2*b) + (8*a^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3
+ (24*a^3*(a^2 - 2*b^2)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^
2 + b^2)*(a^4 - 5*a^2*b^2 + 2*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^6*d
)
```

Maple [A]
time = 0.72, size = 425, normalized size = 1.16

method	result
derivativedivides	$-\frac{b\,a^4}{3\left(a^2+b^2\right)^3\left(a+b\tan(dx+c)\right)^3}+\frac{4ab\left(a^6-9a^4b^2+9a^2b^4-b^6\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^6}-\frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)}{\left(a^2+b^2\right)^5\left(a+b\tan(dx+c)\right)}-\frac{a^3b\left(a^2-2b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(d}$
default	$-\frac{b\,a^4}{3\left(a^2+b^2\right)^3\left(a+b\tan(dx+c)\right)^3}+\frac{4ab\left(a^6-9a^4b^2+9a^2b^4-b^6\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^6}-\frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)}{\left(a^2+b^2\right)^5\left(a+b\tan(dx+c)\right)}-\frac{a^3b\left(a^2-2b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b*a^4/(a^2+b^2)^3/(a+b*tan(d*x+c))^3+4*a*b*(a^6-9*a^4*b^2+9*a^2*b
^4-b^6)/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b
^2)^5/(a+b*tan(d*x+c))-a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+1/(
a^2+b^2)^6*(((-5/8*a^8+15/2*a^6*b^2+5/4*a^4*b^4-13/2*a^2*b^6+3/8*b^8)*tan(d
*x+c)^3+(-4*a^7*b+6*a^5*b^3+8*a^3*b^5-2*a*b^7)*tan(d*x+c)^2+(-3/8*a^8+13/2*
a^6*b^2-15/2*a^2*b^6+5/8*b^8-5/4*a^4*b^4)*tan(d*x+c)-3*a^7*b+7*a^5*b^3+7*a^
3*b^5-3*a*b^7)/(1+tan(d*x+c)^2)^2+1/16*(-32*a^7*b+288*a^5*b^3-288*a^3*b^5+3
2*a*b^7)*ln(1+tan(d*x+c)^2)+1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*a^2*b^6+
3*b^8)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(358) = 716$.
time = 0.61, size = 997, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(b*tan(d*x + c) + a)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
```

```
2) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(a^
12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
 - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*b^5 + 1
03*a^2*b^7 - 3*b^9)*tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 + 165*a^3*
b^6 + 7*a*b^8)*tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^4*b^5 + 6
00*a^2*b^7 - 15*b^9)*tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 822*a^5*b^4
+ 320*a^3*b^6 + 9*a*b^8)*tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*b^3 - 239*a
^4*b^5 + 315*a^2*b^7)*tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 - 423*a^5*b^4
+ 147*a^3*b^6)*tan(d*x + c))/(a^13 + 5*a^11*b^2 + 10*a^9*b^4 + 10*a^7*b^6 +
 5*a^5*b^8 + a^3*b^10 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5
*a^2*b^11 + b^13)*tan(d*x + c)^7 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 1
0*a^5*b^8 + 5*a^3*b^10 + a*b^12)*tan(d*x + c)^6 + (3*a^12*b + 17*a^10*b^3 +
 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^11 + 2*b^13)*tan(d*x + c)^
5 + (a^13 + 11*a^11*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^5*b^8 + 31*a^3*b^1
0 + 6*a*b^12)*tan(d*x + c)^4 + (6*a^12*b + 31*a^10*b^3 + 65*a^8*b^5 + 70*a^
6*b^7 + 40*a^4*b^9 + 11*a^2*b^11 + b^13)*tan(d*x + c)^3 + (2*a^13 + 13*a^11
*b^2 + 35*a^9*b^4 + 50*a^7*b^6 + 40*a^5*b^8 + 17*a^3*b^10 + 3*a*b^12)*tan(d
*x + c)^2 + 3*(a^12*b + 5*a^10*b^3 + 10*a^8*b^5 + 10*a^6*b^7 + 5*a^4*b^9 +
a^2*b^11)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(358) = 716$.
time = 0.48, size = 1053, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*c
os(d*x + c)^7 - 3*(11*a^10*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*a^
2*b^9 + b^11)*cos(d*x + c)^5 - (6*a^10*b + 342*a^8*b^3 - 1830*a^6*b^5 + 614
*a^4*b^7 - 216*a^2*b^9 + 12*b^11 - 3*(3*a^11 - 141*a^9*b^2 + 766*a^7*b^4 -
1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^10)*d*x)*cos(d*x + c)^3 + 3*(114*a^8*b^3
 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^11 + 3*(3*a^9*b^2 - 132*a^7
*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c) + 48*((a^10*
b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*cos(d*x + c)^3 + 3*(a
^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*cos(d*x + c) + (a^7*b^4 - 9*a^5*b
^6 + 9*a^3*b^8 - a*b^10 + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 - 12*a^3*b^8
 + a*b^10)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*b^6 + 105*a^
3*b^8 + 33*a*b^10 + 6*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b
^8 + a*b^10)*cos(d*x + c)^6 - 15*(a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6
- 3*a^3*b^8 - a*b^10)*cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^6*b^5 + 370*a^4
*b^7 - 132*a^2*b^9 + 3*b^11)*d*x + (216*a^9*b^2 - 734*a^7*b^4 + 1590*a^5*b^
```

```
6 - 522*a^3*b^8 - 54*a*b^10 + 3*(9*a^10*b - 399*a^8*b^3 + 1242*a^6*b^5 - 76
6*a^4*b^7 + 141*a^2*b^9 - 3*b^11)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^15
 + 3*a^13*b^2 - 3*a^11*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 39*a^5*b^10 - 17*a^3
*b^12 - 3*a*b^14)*d*cos(d*x + c)^3 + 3*(a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6
+ 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d*cos(d*x + c) + ((3*a^14
*b + 17*a^12*b^3 + 39*a^10*b^5 + 45*a^8*b^7 + 25*a^6*b^9 + 3*a^4*b^11 - 3*a
^2*b^13 - b^15)*d*cos(d*x + c)^2 + (a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20
*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(358) = 716.
time = 0.80, size = 902, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*log(abs(b*tan(d*x + c) +
a))/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^1
1 + b^13) + 3*(24*a^7*b*tan(d*x + c)^4 - 216*a^5*b^3*tan(d*x + c)^4 + 216*a
^3*b^5*tan(d*x + c)^4 - 24*a*b^7*tan(d*x + c)^4 - 5*a^8*tan(d*x + c)^3 + 60
*a^6*b^2*tan(d*x + c)^3 + 10*a^4*b^4*tan(d*x + c)^3 - 52*a^2*b^6*tan(d*x +
c)^3 + 3*b^8*tan(d*x + c)^3 + 16*a^7*b*tan(d*x + c)^2 - 384*a^5*b^3*tan(d*x
 + c)^2 + 496*a^3*b^5*tan(d*x + c)^2 - 64*a*b^7*tan(d*x + c)^2 - 3*a^8*tan(
d*x + c) + 52*a^6*b^2*tan(d*x + c) - 10*a^4*b^4*tan(d*x + c) - 60*a^2*b^6*t
an(d*x + c) + 5*b^8*tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 - 48*a*b^7)/((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)*(tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*tan(d*x + c)^3 - 198*a^5*b^6*tan
(d*x + c)^3 + 198*a^3*b^8*tan(d*x + c)^3 - 22*a*b^10*tan(d*x + c)^3 + 75*a^
8*b^3*tan(d*x + c)^2 - 630*a^6*b^5*tan(d*x + c)^2 + 567*a^4*b^7*tan(d*x + c
```

```
)^2 - 48*a^2*b^9*tan(d*x + c)^2 + 87*a^9*b^2*tan(d*x + c) - 666*a^7*b^4*tan
(d*x + c) + 531*a^5*b^6*tan(d*x + c) - 36*a^3*b^8*tan(d*x + c) + 35*a^10*b
- 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B]
time = 5.66, size = 962, normalized size = 2.63

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{48\,a\,b^3}{{\left(a^2+b^2\right)}^4}+\frac{120\,a\,b^5}{{\left(a^2+b^2\right)}^5}-\frac{80\,a\,b^7}{{\left(a^2+b^2\right)}^6}\right)}{d}-\frac{\frac{2\,\left(11\,a^8\,b-38\,a^6\,b^3+11\,a^4\,b^5\right)}{3\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}-\frac{{\tan(c+d\,x)}^6\,\left(-29\,a^6\,b^3+185\,a^4\,b^5-103\,a^2\,b^7+3\,b^9\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^5\,\left(71\,a^7\,b^2-411\,a^5\,b^4+165\,a^3\,b^6+7\,a\,b^8\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^2\,\left(331\,a^8\,b-1183\,a^6\,b^3-239\,a^4\,b^5+315\,a^2\,b^7\right)}{24\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^3\,\left(5\,a^9+152\,a^7\,b^2-822\,a^5\,b^4+320\,a^3\,b^6+9\,a\,b^8\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}-\frac{{\tan(c+d\,x)}^4\,\left(-149\,a^8\,b+512\,a^6\,b^3+1006\,a^4\,b^5-600\,a^2\,b^7+15\,b^9\right)}{24\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{a\tan(c+d\,x)\,\left(3\,a^8+73\,a^6\,b^2-423\,a^4\,b^4+147\,a^2\,b^6\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(2\,a^3+3\,a\,b^2\right)+{\tan\left(c+d\,x\right)}^5\,\left(3\,a^2\,b+2\,b^3\right)+a^3+{\tan\left(c+d\,x\right)}^4\,\left(a^3+6\,a\,b^2\right)+{\tan\left(c+d\,x\right)}^3\,\left(6\,a^2\,b+b^3\right)+b^3\,{\tan\left(c+d\,x\right)}^7+3\,a\,b^2\,{\tan\left(c+d\,x\right)}^6+3\,a^2\,b\tan\left(c+d\,x\right)\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,14{}\mathrm{i}+3\,b^2\right)}{16\,d\,\left(-a^6\,1{}\mathrm{i}+6\,a^5\,b+a^4\,b^2\,15{}\mathrm{i}-20\,a^3\,b^3-a^2\,b^4\,15{}\mathrm{i}+6\,a\,b^5+b^6\,1{}\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(3\,a^2+a\,b\,14{}\mathrm{i}-3\,b^2\right)}{16\,d\,\left(a^6\,1{}\mathrm{i}+6\,a^5\,b-a^4\,b^2\,15{}\mathrm{i}-20\,a^3\,b^3+a^2\,b^4\,15{}\mathrm{i}+6\,a\,b^5-b^6\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (48*a*b^3)/(a^2 + b^2)^4
+ (120*a*b^5)/(a^2 + b^2)^5 - (80*a*b^7)/(a^2 + b^2)^6))/d - ((2*(11*a^8*b
+ 11*a^4*b^5 - 38*a^6*b^3))/(3*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a
^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)^6*(3*b^9 - 103*a^2*b^7 + 185*a^4*b^5 -
 29*a^6*b^3))/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8
*b^2)) + (tan(c + d*x)^5*(7*a*b^8 + 165*a^3*b^6 - 411*a^5*b^4 + 71*a^7*b^2)
)/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (ta
n(c + d*x)^2*(331*a^8*b + 315*a^2*b^7 - 239*a^4*b^5 - 1183*a^6*b^3))/(24*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^3*(9*a*b^8 + 5*a^9 + 320*a^3*b^6 - 822*a^5*b^4 + 152*a^7*b^2))/(8*(a^10
 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)
^4*(15*b^9 - 149*a^8*b - 600*a^2*b^7 + 1006*a^4*b^5 + 512*a^6*b^3))/(24*(a^
10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a*tan(c +
d*x)*(3*a^8 + 147*a^2*b^6 - 423*a^4*b^4 + 73*a^6*b^2))/(8*(a^10 + b^10 + 5*
a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)))/(d*(tan(c + d*x)^2*(3*a*b^
2 + 2*a^3) + tan(c + d*x)^5*(3*a^2*b + 2*b^3) + a^3 + tan(c + d*x)^4*(6*a*b
^2 + a^3) + tan(c + d*x)^3*(6*a^2*b + b^3) + b^3*tan(c + d*x)^7 + 3*a*b^2*t
an(c + d*x)^6 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) - 1i)*(a*b*14i -
 3*a^2 + 3*b^2))/(16*d*(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i -
 20*a^3*b^3 + a^4*b^2*15i)) - (log(tan(c + d*x) + 1i)*(a*b*14i + 3*a^2 - 3*
b^2))/(16*d*(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3
 - a^4*b^2*15i))
```

3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=264

$$\frac{(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{2\,(a^2+b^2)^5}+\frac{4ab(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}-\frac{a^2b}{3\,(a^2+b^2)^2\,d(a+\dots}$$

```
[Out] 1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+4*a*b*(a^4-5*a^2*b^2+2*
b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/3*a^2*b/(a^2+b^2)^2/d/(a
+b*tan(d*x+c))^3-a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2-b*(3*a^4-8*
a^2*b^2+b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(4*a*b*(a^2-b^
2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A]
time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$-\frac{a^2b}{3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^3}-\frac{ab(a^2-b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2}-\frac{b(3a^4-8a^2b^2+b^4)}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{2d\,(a^2+b^2)^4}+\frac{4ab(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5}+\frac{x(a^6-25a^4b^2+35a^2b^4-3b^6)}{2\,(a^2+b^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(2*(a^2 + b^2)^5) + (4*a*b*(a^4
 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*
d) - (a^2*b)/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^3) - (a*b*(a^2 - b^2))
/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 - 8*a^2*b^2 + b^4))/(
(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(4*a*b*(a^2 - b^2)
+ (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^4(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2}{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \left(-\frac{\cdots}{(a\cdots}\right.\right.}{\cdots}$$

$$= \frac{4ab(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx)\cdots}$$

$$= \frac{4ab(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx)\cdots}$$

$$= \frac{(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{2\left(a^2+b^2\right)^5} + \frac{4ab(a^4-5a^2b^2+2b^4)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{4ab\cdots}{\cdots}$$

Mathematica [A]
time = 3.96, size = 395, normalized size = 1.50

$$-\frac{b\left(\frac{3(a^2+b^2)(a^4-6a^2b^2+b^4)\text{ArcTan}(\tan(c+dx))}{b}+12a(a-b)(a+b)\left(a^2+b^2\right)\cos^2(c+dx)+3\left(4a^5-20a^3b^2+8ab^4+\frac{-a^6+15a^4b^2-15a^2b^4+b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-24a(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))+3\left(4a^5-20a^3b^2+8ab^4+\frac{a^6-15a^4b^2+15a^2b^4-b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+\frac{3(a^2+b^2)(a^4-6a^2b^2+b^4)\sin(2(c+dx))}{2b}+\frac{2a^2(a^2+b^2)^3}{(a+b\tan(c+dx))^3}+\frac{6a(a-b)(a+b)(a^2+b^2)^2}{(a+b\tan(c+dx))^2}+\frac{6(a^2+b^2)(3a^4-8a^2b^2+b^4)}{a+b\tan(c+dx)}\right)}{6\left(a^2+b^2\right)^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 1
2*a*(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 + 8*
a*b^4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]] +
 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*a^2*b^
2 + b^4)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*Tan[c + d*x
])^3 + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (6*(a^2
 + b^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^5*d)
```

Maple [A]
time = 0.61, size = 288, normalized size = 1.09

method	result

derivativedivides	$-\frac{a^2b}{3\left(a^2+b^2\right)^2(a+b\tan(dx+c))^3}-\frac{b\left(3a^4-8a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}-\frac{ab\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}+\frac{4ab\left(a^4-5a^2b^2+2b^4\right)\ln(a+b\tan(dx+}{\left(a^2+b^2\right)^5}$
default	$-\frac{a^2b}{3\left(a^2+b^2\right)^2(a+b\tan(dx+c))^3}-\frac{b\left(3a^4-8a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}-\frac{ab\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}+\frac{4ab\left(a^4-5a^2b^2+2b^4\right)\ln(a+b\tan(dx+}{\left(a^2+b^2\right)^5}$
risch	$-\frac{3ixb}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{xa}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-4ia^3b+4ia\,b^3+a^4-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^3-b*(3*a^4-8*a^2*b^2+b^4)/(a^2
+b^2)^4/(a+b*tan(d*x+c))-a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))^2+4*a*b
*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((-1/2
*a^6+5/2*a^4*b^2+5/2*a^2*b^4-1/2*b^6)*tan(d*x+c)-2*a^5*b+2*a*b^5)/(1+tan(d*
x+c)^2)+1/4*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1+tan(d*x+c)^2)+1/2*(a^6-25*a
^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(258) = 516$.
time = 0.56, size = 662, normalized size = 2.51

$$\frac{\frac{3\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(a^5b-5a^3b^3+2ab^5\right)\log(b\tan(dx+c)+a)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(a^5b-5a^3b^3+2ab^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{38a^6b-56a^4b^3+2a^2b^5+3\left(7a^4b^3-22a^2b^5+3b^7\right)\tan(dx+c)^4+3\left(17a^5b^2-46a^3b^4+ab^6\right)\tan(dx+c)^3+\left(35a^6b-44a^4b^3-73a^2b^5+6b^7\right)\tan(dx+c)^2+3\left(a^7+20a^5b^2-43a^3b^4+2ab^6\right)\tan(dx+c)}{a^{11}+4a^9b^2+6a^7b^4+4a^5b^6+a^3b^8+\left(a^8b^3+4a^6b^5+6a^4b^7+4a^2b^9+b^{11}\right)\tan(dx+c)^5+3\left(a^9b^2+4a^7b^4+6a^5b^6+4a^3b^8+ab^{10}\right)\tan(dx+c)^4+\left(3a^{10}b+13a^8b^3+22a^6b^5+18a^4b^7+7a^2b^9+b^{11}\right)\tan(dx+c)^3+\left(a^{11}+7a^9b^2+18a^7b^4+22a^5b^6+13a^3b^8+3ab^{10}\right)\tan(dx+c)^2+3\left(a^{10}b+4a^8b^3+6a^6b^5+4a^4b^7+a^2b^9\right)\tan(dx+c)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*log(tan(d*x + c)^2 +
1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (38*a^
6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*tan(d*x +
 c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*tan(d*x + c)^3 + (35*a^6*b - 44
*a^4*b^3 - 73*a^2*b^5 + 6*b^7)*tan(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2 - 43*a^
3*b^4 + 2*a*b^6)*tan(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 +
a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)
^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^
4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^9 + b^11)*ta
n(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3
*a*b^10)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a
^2*b^9)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(258) = 516$.
time = 0.46, size = 802, normalized size = 3.04

$$-\frac{3\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\cos(dx+c)^5+(3\,a^8b+111\,a^6b^3-231\,a^4b^5+65\,a^2b^7-12\,b^9-3\,(a^9-28\,a^7b^2+110\,a^5b^4-108\,a^3b^6+9\,ab^8)dx)\cos(dx+c)^3-3\,(25\,a^6b^3-51\,a^4b^5+25\,a^2b^7-3\,b^9+3\,(a^7b^2-25\,a^5b^4+35\,a^3b^6-3\,ab^8)dx)\cos(dx+c)-12\,((a^8b-8\,a^6b^3+17\,a^4b^5-6\,a^2b^7)\cos(dx+c)^3+3\,(a^6b^3-5\,a^4b^5+2\,a^2b^7)\cos(dx+c)+(a^5b^4-5\,a^3b^6+2\,ab^8+(3\,a^7b^2-16\,a^5b^4+11\,a^3b^6-2\,ab^8)\cos(dx+c)^2)\sin(dx+c))\log(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2)-(32\,a^5b^4-66\,a^3b^6+6\,ab^8-3\,(a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)\cos(dx+c)^4+3\,(a^6b^3-25\,a^4b^5+35\,a^2b^7-3\,b^9)dx+(45\,a^7b^2-143\,a^5b^4+219\,a^3b^6-9\,ab^8+3\,(3\,a^8b-76\,a^6b^3+130\,a^4b^5-44\,a^2b^7+3\,b^9)dx)\cos(dx+c)^2)\sin(dx+c)}{6\,((a^{13}+2\,a^{11}b^2-5\,a^9b^4-20\,a^7b^6-25\,a^5b^8-14\,a^3b^{10}-3\,ab^{12})d\cos(dx+c)^3+3\,(a^{11}b^2+5\,a^9b^4+10\,a^7b^6+10\,a^5b^8+5\,a^3b^{10}+ab^{12})d\cos(dx+c)+((3\,a^{12}b+14\,a^{10}b^3+25\,a^8b^5+20\,a^6b^7+5\,a^4b^9-2\,a^2b^{11}-b^{13})d\cos(dx+c)^2+(a^{10}b^3+5\,a^8b^5+10\,a^6b^7+10\,a^4b^9+5\,a^2b^{11}+b^{13})d)\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 +
(3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*a^
7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*cos(d*x + c)^3 - 3*(25*a^
6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*
b^6 - 3*a*b^8)*d*x)*cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5 - 6*
a^2*b^7)*cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*cos(d*x + c)
+ (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*b^6 - 2
*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*a*b^8 - 3
*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^4 + 3*(a^6*
b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a^5*b^4 + 21
9*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 44*a^2*b^7 +
3*b^9)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^13 + 2*a^11*b^2 - 5*a^9*b^4 -
 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*cos(d*x + c)^3 + 3*(a^
11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*d*cos(d
*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 -
2*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 +
10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(258) = 516$.
time = 0.79, size = 642, normalized size = 2.43

$$\frac{\frac{3\,(a^6-25\,a^4b^2+35\,a^2b^4-3\,b^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{12\,(a^5b-5\,a^3b^3+2\,ab^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{24\,(a^5b^2-5\,a^3b^4+2\,ab^6)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}+\frac{3\left(4\,a^5b\tan(dx+c)^2-20\,a^3b^3\tan(dx+c)^2+8\,ab^5\tan(dx+c)^2-a^6\tan(dx+c)+5\,a^4b^2\tan(dx+c)+5\,a^2b^4\tan(dx+c)-b^6\tan(dx+c)-20\,a^3b^3+12\,ab^5\right)}{(a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10})\left(\tan(dx+c)^2+1\right)}-\frac{2\left(22\,a^5b^4\tan(dx+c)^3-110\,a^3b^6\tan(dx+c)^3+44\,ab^8\tan(dx+c)^3+75\,a^6b^3\tan(dx+c)^2-345\,a^4b^5\tan(dx+c)^2+111\,a^2b^7\tan(dx+c)^2+3\,b^9\tan(dx+c)^2+87\,a^7b^2\tan(dx+c)-357\,a^5b^4\tan(dx+c)+87\,a^3b^6\tan(dx+c)+3\,ab^8\tan(dx+c)+35\,a^8b-119\,a^6b^3+23\,a^4b^5+a^2b^7\right)}{(a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10})(b\tan(dx+c)+a)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*log(abs(b*tan(d*x +
 c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)
 + 3*(4*a^5*b*tan(d*x + c)^2 - 20*a^3*b^3*tan(d*x + c)^2 + 8*a*b^5*tan(d*x
+ c)^2 - a^6*tan(d*x + c) + 5*a^4*b^2*tan(d*x + c) + 5*a^2*b^4*tan(d*x + c)
 - b^6*tan(d*x + c) - 20*a^3*b^3 + 12*a*b^5)/((a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)) - 2*(22*a^5*b^4*ta
n(d*x + c)^3 - 110*a^3*b^6*tan(d*x + c)^3 + 44*a*b^8*tan(d*x + c)^3 + 75*a^
6*b^3*tan(d*x + c)^2 - 345*a^4*b^5*tan(d*x + c)^2 + 111*a^2*b^7*tan(d*x + c
)^2 + 3*b^9*tan(d*x + c)^2 + 87*a^7*b^2*tan(d*x + c) - 357*a^5*b^4*tan(d*x
+ c) + 87*a^3*b^6*tan(d*x + c) + 3*a*b^8*tan(d*x + c) + 35*a^8*b - 119*a^6*
b^3 + 23*a^4*b^5 + a^2*b^7)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B]

time = 5.09, size = 597, normalized size = 2.26

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{28\,a\,b^3}{{\left(a^2+b^2\right)}^4}+\frac{32\,a\,b^5}{{\left(a^2+b^2\right)}^5}\right)}{d}-\frac{\frac{{\tan(c+d\,x)}^2\,\left(35\,a^4\,b-79\,a^2\,b^3+6\,b^5\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^4\,\left(7\,a^4\,b^3-22\,a^2\,b^5+3\,b^7\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{{\tan(c+d\,x)}^3\,\left(17\,a^5\,b^2-46\,a^3\,b^4+a\,b^6\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{a^2\,\left(19\,a^4\,b-28\,a^2\,b^3+b^5\right)}{3\,\left(a^2+b^2\right)\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{a\tan(c+d\,x)\,\left(a^6+20\,a^4\,b^2-43\,a^2\,b^4+2\,b^6\right)}{2\,\left(a^2+b^2\right)\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^3+{\tan\left(c+d\,x\right)}^2\,\left(a^3+3\,a\,b^2\right)+{\tan\left(c+d\,x\right)}^3\,\left(3\,a^2\,b+b^3\right)+b^3\,{\tan\left(c+d\,x\right)}^5+3\,a\,b^2\,{\tan\left(c+d\,x\right)}^4+3\,a^2\,b\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(a^5+a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4+b^5\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(-3\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(a^5-a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4-b^5\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (28*a*b^3)/(a^2 + b^2)^4
+ (32*a*b^5)/(a^2 + b^2)^5))/d - ((tan(c + d*x)^2*(35*a^4*b + 6*b^5 - 79*a^
2*b^3))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^4*(3*b^7 -
22*a^2*b^5 + 7*a^4*b^3))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)
) + (tan(c + d*x)^3*(a*b^6 - 46*a^3*b^4 + 17*a^5*b^2))/(2*(a^8 + b^8 + 4*a^
2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a^2*(19*a^4*b + b^5 - 28*a^2*b^3))/(3*(a
^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)*(a^6 + 2*b
^6 - 43*a^2*b^4 + 20*a^4*b^2))/(2*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)))/(d*(a^3 + tan(c + d*x)^2*(3*a*b^2 + a^3) + tan(c + d*x)^3*(3*a^2*b
 + b^3) + b^3*tan(c + d*x)^5 + 3*a*b^2*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x
))) - (log(tan(c + d*x) - 1i)*(a*1i + 3*b))/(4*d*(5*a*b^4 + a^4*b*5i + a^5
+ b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (log(tan(c + d*x) + 1i)*(a*1i - 3*b
))/(4*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$-\frac{\cot(c+dx)}{a^4d}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan(c+dx))}{a^5d}-\frac{b}{3a^2d(a+b\tan(c+dx))^3}-\frac{b}{a^3d(a+b\tan(c+$$

```
[Out] -cot(d*x+c)/a^4/d-4*b*ln(tan(d*x+c))/a^5/d+4*b*ln(a+b*tan(d*x+c))/a^5/d-1/3
*b/a^2/d/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-3*b/a^4/d/(a+b*tan(d
*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 46}

$$-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan(c+dx))}{a^5d}-\frac{3b}{a^4d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^4d}-\frac{b}{a^3d(a+b\tan(c+dx))^2}-\frac{b}{3a^2d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(Cot[c + d*x]/(a^4*d)) - (4*b*Log[Tan[c + d*x]])/(a^5*d) + (4*b*Log[a + b*
Tan[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*Tan[c + d*x])^3) - b/(a^3*d*(a +
 b*Tan[c + d*x])^2) - (3*b)/(a^4*d*(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right)\,dx, x, b\tan(c\right.}{d}$$
$$= -\frac{\cot(c+dx)}{a^4d} - \frac{4b\log(\tan(c+dx))}{a^5d} + \frac{4b\log(a+b\tan(c+dx))}{a^5d} - \frac{}{3a^2d(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(116) = 232$.
time = 2.34, size = 259, normalized size = 2.23

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-3a(b+a\cot(c+dx))^3\sin^2(c+dx)+\frac{a^2b^4\tan(c+dx)}{a^2+b^2}+\frac{b^2(18a^4+23a^2b^2+9b^4)(a\cos(c+dx)+b\sin(c+dx))^2\tan(c+dx)}{(a^2+b^2)^2}-\frac{2a^2b^3(3a^2+2b^2)(a+b\tan(c+dx))}{(a^2+b^2)^2}-12b\cos^2(c+dx)\log(\sin(c+dx))(a+b\tan(c+dx))^3+12b\cos^2(c+dx)\log(a\cos(c+dx)+b\sin(c+dx))(a+b\tan(c+dx))^3\right)}{3a^5d(a+b\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-3*a*(b + a*Cot[c + d*x]
)^3*Sin[c + d*x]^2 + (a^2*b^4*Tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 + 23
*a^2*b^2 + 9*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 +
b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*Tan[c + d*x]))/(a^2 + b^2)^2 - 1
2*b*Cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*Tan[c + d*x])^3 + 12*b*Cos[c +
d*x]^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a + b*Tan[c + d*x])^3))/(3*a^5
*d*(a + b*Tan[c + d*x])^4)
```

Maple [A]
time = 0.42, size = 103, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}}{d}$
default	$\frac{-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}}{d}$
risch	$-\frac{2i(18ia^5b\,e^{6i(dx+c)}+18ia^5b\,e^{2i(dx+c)}+36ia^5b\,e^{4i(dx+c)}-3a^6e^{6i(dx+c)}+63a^4b^2e^{6i(dx+c)}-120a^2b^4e^{6i(dx+c)}+12b^6e^{6i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b/a^2/(a+b*tan(d*x+c))^3+4/a^5*b*ln(a+b*tan(d*x+c))-3/a^4*b/(a+b*
tan(d*x+c))-1/a^3*b/(a+b*tan(d*x+c))^2-1/a^4/tan(d*x+c)-4/a^5*b*ln(tan(d*x+
c)))
```

Maxima [A]
time = 0.31, size = 140, normalized size = 1.21

$$-\frac{\frac{12\,b^3\tan(dx+c)^3+30\,ab^2\tan(dx+c)^2+22\,a^2b\tan(dx+c)+3\,a^3}{a^4b^3\tan(dx+c)^4+3\,a^5b^2\tan(dx+c)^3+3\,a^6b\tan(dx+c)^2+a^7\tan(dx+c)}-\frac{12\,b\log(b\tan(dx+c)+a)}{a^5}+\frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x +
 c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b*t
an(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 12*b
*log(tan(d*x + c))/a^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(114) = 228.
time = 0.41, size = 874, normalized size = 7.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*b
^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 - 10
2*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8
 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (3*a^
8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + ((a^9*
b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^
5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^4*b^6 +
3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^
4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2
 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 +
 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*b^7 - 12*a*b^
9)*cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*a*b^9)*cos(d*x
 + c))*sin(d*x + c))/((3*a^13*b + 8*a^11*b^3 + 6*a^9*b^5 - a^5*b^9)*d*cos(d
*x + c)^4 - (3*a^13*b + 7*a^11*b^3 + 3*a^9*b^5 - 3*a^7*b^7 - 2*a^5*b^9)*d*c
os(d*x + c)^2 - (a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*d - ((a^14 - 6
*a^10*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*cos(d*x + c)^3 + 3*(a^12*b^2 + 3*a^10*
b^4 + 3*a^8*b^6 + a^6*b^8)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**4, x)
```

Giac [A]
time = 0.66, size = 129, normalized size = 1.11

$$\frac{\frac{12\,b\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^5}-\frac{12\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^5}+\frac{3\,\left(4\,b\tan\left(dx+c\right)-a\right)}{a^5\tan\left(dx+c\right)}-\frac{22\,b^4\tan\left(dx+c\right)^3+75\,ab^3\tan\left(dx+c\right)^2+87\,a^2b^2\tan\left(dx+c\right)+35\,a^3b}{\left(b\tan\left(dx+c\right)+a\right)^3a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a^
5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3 +
75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*x +
 c) + a)^3*a^5))/d
```

Mupad [B]
time = 3.98, size = 131, normalized size = 1.13

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\tan\left(c+d\,x\right)}{a}+1\right)}{a^5\,d}-\frac{\frac{1}{a}+\frac{10\,b^2\tan\left(c+d\,x\right)^2}{a^3}+\frac{4\,b^3\tan\left(c+d\,x\right)^3}{a^4}+\frac{22\,b\tan\left(c+d\,x\right)}{3\,a^2}}{d\,\left(a^3\tan\left(c+d\,x\right)+3\,a^2\,b\tan\left(c+d\,x\right)^2+3\,a\,b^2\tan\left(c+d\,x\right)^3+b^3\tan\left(c+d\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^5*d) - (1/a + (10*b^2*tan(c + d*x)
^2)/a^3 + (4*b^3*tan(c + d*x)^3)/a^4 + (22*b*tan(c + d*x))/(3*a^2))/(d*(a^3
*tan(c + d*x) + b^3*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*tan(c
 + d*x)^3))
```

3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=205

$$-\frac{(a^2+10b^2)\cot(c+dx)}{a^6d}+\frac{2b\cot^2(c+dx)}{a^5d}-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d}+\frac{4b(a^2+5b^2)\log(a}{a^7}$$

```
[Out] -(a^2+10*b^2)*cot(d*x+c)/a^6/d+2*b*cot(d*x+c)^2/a^5/d-1/3*cot(d*x+c)^3/a^4/
d-4*b*(a^2+5*b^2)*ln(tan(d*x+c))/a^7/d+4*b*(a^2+5*b^2)*ln(a+b*tan(d*x+c))/a
^7/d-1/3*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^3-b*(a^2+2*b^2)/a^5/d/(a+b*tan(
d*x+c))^2-b*(3*a^2+10*b^2)/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{2b\cot^2(c+dx)}{a^5d}-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d}+\frac{4b(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7d}-\frac{b(3a^2+10b^2)}{a^6d(a+b\tan(c+dx))}-\frac{(a^2+10b^2)\cot(c+dx)}{a^6d}-\frac{b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))^2}-\frac{b(a^2+b^2)}{3a^4d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - C
ot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (
4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4
*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^
2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^4x^4} - \frac{4b^2}{a^5x^3} + \frac{a^2+10b^2}{a^6x^2} - \frac{4(a^2+5b^2)}{a^7x} + \frac{a^2+b^2}{a^4(a+x)^4} + \frac{2(a^2+2b^2)}{a^5(a+x)^3} + \frac{3a^2+10b^2}{a^6(a+x)^2} + \cdots\right.\right.}{d}$$

$$= -\frac{(a^2+10b^2)\cot(c+dx)}{a^6d} + \frac{2b\cot^2(c+dx)}{a^5d} - \frac{\cot^3(c+dx)}{3a^4d} - \frac{4b(a^2+5b^2)\log\cdots}{a^7\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 528 vs. 2(205) = 410.
time = 2.21, size = 528, normalized size = 2.58

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-192b(a^2+5b^2)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3+192b(a^2+5b^2)\log(a\cos(c+dx)+b\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3-\frac{\csc^3(c+dx)(8a^8-4a^6b^2-50a^4b^4-190a^2b^6-150b^8+3(3a^8+10a^6b^2+45a^4b^4+115a^2b^6+75b^8)\cos(2(c+dx))+6(2a^6b^2-17a^4b^4-35a^2b^6-15b^8)\cos(4(c+dx))-a^8\cos(6(c+dx))-22a^6b^2\cos(6(c+dx))+17a^4b^4\cos(6(c+dx))+55a^2b^6\cos(6(c+dx))+15b^8\cos(6(c+dx))-3a^7b\sin(2(c+dx))+3a^5b^3\sin(2(c+dx))-75a^3b^5\sin(2(c+dx))-75ab^7\sin(2(c+dx))-6a^7b\sin(4(c+dx))+84a^5b^3\sin(4(c+dx))+156a^3b^5\sin(4(c+dx))+60ab^7\sin(4(c+dx))-3a^7b\sin(6(c+dx))-65a^5b^3\sin(6(c+dx))-79a^3b^5\sin(6(c+dx))-15ab^7\sin(6(c+dx)))}{a^2+b^2}\right)}{48a^7d(a+b\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Log
[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (C
sc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 + 3*(
3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x)] +
6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a^8*Cos
[6*(c + d*x)] - 22*a^6*b^2*Cos[6*(c + d*x)] + 17*a^4*b^4*Cos[6*(c + d*x)] +
 55*a^2*b^6*Cos[6*(c + d*x)] + 15*b^8*Cos[6*(c + d*x)] - 3*a^7*b*Sin[2*(c +
 d*x)] + 3*a^5*b^3*Sin[2*(c + d*x)] - 75*a^3*b^5*Sin[2*(c + d*x)] - 75*a*b^
7*Sin[2*(c + d*x)] - 6*a^7*b*Sin[4*(c + d*x)] + 84*a^5*b^3*Sin[4*(c + d*x)]
 + 156*a^3*b^5*Sin[4*(c + d*x)] + 60*a*b^7*Sin[4*(c + d*x)] - 3*a^7*b*Sin[6
*(c + d*x)] - 65*a^5*b^3*Sin[6*(c + d*x)] - 79*a^3*b^5*Sin[6*(c + d*x)] - 1
5*a*b^7*Sin[6*(c + d*x)]))/(a^2 + b^2)))/(48*a^7*d*(a + b*Tan[c + d*x])^4)
```

Maple [A]
time = 0.46, size = 184, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{b(3a^2+10b^2)}{a^6(a+b\tan(dx+c))}-\frac{(a^2+b^2)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b(a^2+2b^2)}{a^5(a+b\tan(dx+c))^2}+\frac{4b(a^2+5b^2)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10\cdots}{a^6\tan(d\cdots}}{d}$
default	$\frac{-\frac{b(3a^2+10b^2)}{a^6(a+b\tan(dx+c))}-\frac{(a^2+b^2)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b(a^2+2b^2)}{a^5(a+b\tan(dx+c))^2}+\frac{4b(a^2+5b^2)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10\cdots}{a^6\tan(d\cdots}}{d}$

risch	$-\frac{4i\left(-a^6b-26a^4b^3+420a^2b^5\mathrm{e}^{8i(dx+c)}+270a^2b^5\mathrm{e}^{2i(dx+c)}-6a^4b^3\mathrm{e}^{10i(dx+c)}-174a^2b^5\mathrm{e}^{10i(dx+c)}-3a^6b\,\mathrm{e}^{8i(dx+c)}-3ia^7\mathrm{e}^{8i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*(3*a^2+10*b^2)/a^6/(a+b*tan(d*x+c))-1/3*(a^2+b^2)*b/a^4/(a+b*tan(d*
x+c))^3-b*(a^2+2*b^2)/a^5/(a+b*tan(d*x+c))^2+4*b*(a^2+5*b^2)/a^7*ln(a+b*tan
(d*x+c))-1/3/a^4/tan(d*x+c)^3-(a^2+10*b^2)/a^6/tan(d*x+c)+2/a^5*b/tan(d*x+c
)^2-4*b*(a^2+5*b^2)/a^7*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.32, size = 228, normalized size = 1.11

$$\frac{\frac{3\,a^4b\tan(dx+c)-12\,(a^2b^3+5\,b^5)\tan(dx+c)^5-a^5-30\,(a^3b^2+5\,ab^4)\tan(dx+c)^4-22\,(a^4b+5\,a^2b^3)\tan(dx+c)^3-3\,(a^5+5\,a^3b^2)\tan(dx+c)^2}{a^6b^3\tan(dx+c)^6+3\,a^7b^2\tan(dx+c)^5+3\,a^8b\tan(dx+c)^4+a^9\tan(dx+c)^3}+\frac{12\,(a^2b+5\,b^3)\log(b\tan(dx+c)+a)}{a^7}-\frac{12\,(a^2b+5\,b^3)\log(\tan(dx+c))}{a^7}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 30
*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)^3
 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b^2*
tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2*b +
 5*b^3)*log(b*tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*log(tan(d*x + c))/
a^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(201) = 402$.
time = 0.43, size = 1235, normalized size = 6.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^10 + 23*a^8*b^2 - 22*a^6*b
^4 - 138*a^4*b^6 - 90*a^2*b^8)*cos(d*x + c)^6 - 3*(a^10 + 25*a^8*b^2 - 46*a
^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a^6*
b^4 - 131*a^4*b^6 - 80*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6 + 1
1*a^2*b^8 + 5*b^10 + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b
^10)*cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*
b^10)*cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b
^10)*cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*
b^9)*cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9
```

```
)*cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x +
 c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^10 + (3*a^8*b^2
 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^10)*cos(d*x + c)^6 - 3*(2*a^8*
b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^10)*cos(d*x + c)^4 + 3*(a^8*b
^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^10)*cos(d*x + c)^2 - ((a^9*b +
 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*cos(d*x + c)^5 - (a^9*b +
a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*cos(d*x + c)^3 - 3*(a^7*b^3 +
 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos
(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*b^5 + 34*a^3*b^7 -
30*a*b^9)*cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + 350*a^5*b^5 + 26*a^3*b^
7 - 120*a*b^9)*cos(d*x + c)^3 + 3*(15*a^7*b^3 + 23*a^5*b^5 - 14*a^3*b^7 - 2
0*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 5*a^11*b^3 + a^9*b^5 - a^
7*b^7)*d*cos(d*x + c)^6 - 3*(2*a^13*b + 3*a^11*b^3 - a^7*b^7)*d*cos(d*x + c
)^4 + 3*(a^13*b + a^11*b^3 - a^9*b^5 - a^7*b^7)*d*cos(d*x + c)^2 + (a^11*b^
3 + 2*a^9*b^5 + a^7*b^7)*d - ((a^14 - a^12*b^2 - 5*a^10*b^4 - 3*a^8*b^6)*d*
cos(d*x + c)^5 - (a^14 - 4*a^12*b^2 - 11*a^10*b^4 - 6*a^8*b^6)*d*cos(d*x +
c)^3 - 3*(a^12*b^2 + 2*a^10*b^4 + a^8*b^6)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**4, x)
```

Giac [A]
time = 0.68, size = 222, normalized size = 1.08

$$-\frac{\frac{12\,(a^2b+5\,b^3)\log(|\tan(dx+c)|)}{a^7}-\frac{12\,(a^2b^2+5\,b^4)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{12\,a^2b^3\tan(dx+c)^5+60\,b^5\tan(dx+c)^5+30\,a^3b^2\tan(dx+c)^4+150\,ab^4\tan(dx+c)^4+22\,a^4b\tan(dx+c)^3+110\,a^2b^3\tan(dx+c)^3+3\,a^5\tan(dx+c)^2+15\,a^3b^2\tan(dx+c)^2-3\,a^4b\tan(dx+c)+a^5}{\left(b\tan(dx+c)^2+a\tan(dx+c)\right)^3a^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)*
log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^5*
tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 + 22*
a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^2 +
15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x + c)^2
+ a*tan(d*x + c))^3*a^6))/d
```

Mupad [B]
time = 5.06, size = 232, normalized size = 1.13

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,\left(a^2+5\,b^2\right)\,\left(a+2\,b\,\tan\left(c+d\,x\right)\right)}{a\,\left(4\,a^2\,b+20\,b^3\right)}\right)\,\left(a^2+5\,b^2\right)}{a^7\,d}-\frac{\frac{1}{3\,a}+\frac{\tan\left(c+d\,x\right)^2\,\left(a^2+5\,b^2\right)}{a^3}-\frac{b\,\tan\left(c+d\,x\right)}{a^2}+\frac{22\,b\,\tan\left(c+d\,x\right)^3\,\left(a^2+5\,b^2\right)}{3\,a^4}+\frac{10\,b^2\,\tan\left(c+d\,x\right)^4\,\left(a^2+5\,b^2\right)}{a^5}+\frac{4\,b^3\,\tan\left(c+d\,x\right)^5\,\left(a^2+5\,b^2\right)}{a^6}}{d\,\left(a^3\,\tan\left(c+d\,x\right)^3+3\,a^2\,b\,\tan\left(c+d\,x\right)^4+3\,a\,b^2\,\tan\left(c+d\,x\right)^5+b^3\,\tan\left(c+d\,x\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a^2 + 5*b^2)*(a + 2*b*tan(c + d*x)))/(a*(4*a^2*b + 20*b^3)
))*(a^2 + 5*b^2))/(a^7*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 5*b^2))/a^3 -
 (b*tan(c + d*x))/a^2 + (22*b*tan(c + d*x)^3*(a^2 + 5*b^2))/(3*a^4) + (10*b
^2*tan(c + d*x)^4*(a^2 + 5*b^2))/a^5 + (4*b^3*tan(c + d*x)^5*(a^2 + 5*b^2))
/a^6)/(d*(a^3*tan(c + d*x)^3 + b^3*tan(c + d*x)^6 + 3*a^2*b*tan(c + d*x)^4
+ 3*a*b^2*tan(c + d*x)^5))
```

3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=300

$$-\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d}+\frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d}-\frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d}+\frac{b\cot^4(c+dx)}{a^5d}-$$

```
[Out] -(a^4+20*a^2*b^2+35*b^4)*cot(d*x+c)/a^8/d+2*b*(2*a^2+5*b^2)*cot(d*x+c)^2/a^
7/d-2/3*(a^2+5*b^2)*cot(d*x+c)^3/a^6/d+b*cot(d*x+c)^4/a^5/d-1/5*cot(d*x+c)^
5/a^4/d-4*b*(a^4+10*a^2*b^2+14*b^4)*ln(tan(d*x+c))/a^9/d+4*b*(a^4+10*a^2*b^
2+14*b^4)*ln(a+b*tan(d*x+c))/a^9/d-1/3*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
^3-b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))^2-b*(3*a^4+20*a^2*b^2+21*
b^4)/a^8/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 0.21, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\frac{b\cot^4(c+dx)}{a^5d}-\frac{\cot^5(c+dx)}{5a^4d}-\frac{b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))^2}+\frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d}-\frac{b(a^2+b^2)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d}-\frac{4b(a^4+10a^2b^2+14b^4)\log(\tan(c+dx))}{a^9d}+\frac{4b(a^4+10a^2b^2+14b^4)\log(a+b\tan(c+dx))}{a^9d}-\frac{b(3a^4+20a^2b^2+21b^4)}{a^8d(a+b\tan(c+dx))}-\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2
)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b
*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*L
og[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c +
 d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) -
(b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^4x^6} - \frac{4b^4}{a^5x^5} + \frac{2b^2\left(a^2+5b^2\right)}{a^6x^4} - \frac{4\left(2a^2b^2+5b^4\right)}{a^7x^3} + \frac{a^4+20a^2b^2+35b^4}{a^8x^2} - \frac{4\left(a^4+10a^2b^2+\right.}{a^9x}\right.\right.}{}$$

$$= -\frac{\left(a^4+20a^2b^2+35b^4\right)\cot(c+dx)}{a^8d} + \frac{2b\left(2a^2+5b^2\right)\cot^2(c+dx)}{a^7d} - \frac{2\left(a^2+5b^2\right)}{3a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 673 vs. 2(300) = 600.
time = 1.82, size = 673, normalized size = 2.24

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 7680*b*
(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 3070*a
^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^4*b^4
 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2 + 1010
*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*(c + d*
x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)] - 8540*a
^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*(c + d*x)
] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] + 1645*a^2*b
^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*Sin[2*(c + d*x)]
 + 372*a^5*b^3*Sin[2*(c + d*x)] + 4830*a^3*b^5*Sin[2*(c + d*x)] + 1470*a*b^
7*Sin[2*(c + d*x)] + 144*a^7*b*Sin[4*(c + d*x)] - 2476*a^5*b^3*Sin[4*(c + d
*x)] - 9730*a^3*b^5*Sin[4*(c + d*x)] - 1470*a*b^7*Sin[4*(c + d*x)] - 24*a^7
*b*Sin[6*(c + d*x)] + 2756*a^5*b^3*Sin[6*(c + d*x)] + 7670*a^3*b^5*Sin[6*(c
 + d*x)] + 630*a*b^7*Sin[6*(c + d*x)] - 24*a^7*b*Sin[8*(c + d*x)] - 922*a^5
*b^3*Sin[8*(c + d*x)] - 2095*a^3*b^5*Sin[8*(c + d*x)] - 105*a*b^7*Sin[8*(c
+ d*x)])))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)
```

Maple [A]
time = 0.50, size = 280, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{b(3a^4+20a^2b^2+21b^4)}{a^8(a+b\tan(dx+c))}-\frac{(a^4+2a^2b^2+b^4)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b(a^4+4a^2b^2+3b^4)}{a^7(a+b\tan(dx+c))^2}+\frac{4b(a^4+10a^2b^2+14b^4)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c}}{d}$
default	$\frac{-\frac{b(3a^4+20a^2b^2+21b^4)}{a^8(a+b\tan(dx+c))}-\frac{(a^4+2a^2b^2+b^4)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b(a^4+4a^2b^2+3b^4)}{a^7(a+b\tan(dx+c))^2}+\frac{4b(a^4+10a^2b^2+14b^4)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8/(a+b*tan(d*x+c))-1/3*(a^4+2*a^2*b^2+b
^4)*b/a^6/(a+b*tan(d*x+c))^3-b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+c))^2
+4*b*(a^4+10*a^2*b^2+14*b^4)/a^9*ln(a+b*tan(d*x+c))-1/5/a^4/tan(d*x+c)^5-1/
3*(2*a^2+10*b^2)/a^6/tan(d*x+c)^3-(a^4+20*a^2*b^2+35*b^4)/a^8/tan(d*x+c)+1/
a^5*b/tan(d*x+c)^4+2*b*(2*a^2+5*b^2)/a^7/tan(d*x+c)^2-4*b*(a^4+10*a^2*b^2+1
4*b^4)/a^9*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.36, size = 325, normalized size = 1.08

$$\frac{\frac{6\,a^6 b\tan(dx+c)-60\,(a^4b^3+10\,a^2b^5+14\,b^7)\tan(dx+c)^7-3\,a^7-150\,(a^5b^2+10\,a^3b^4+14\,ab^6)\tan(dx+c)^6-110\,(a^6b+10\,a^4b^3+14\,a^2b^5)\tan(dx+c)^5-15\,(a^7+10\,a^5b^2+14\,a^3b^4)\tan(dx+c)^4+6\,(5\,a^6b+7\,a^4b^3)\tan(dx+c)^3-2\,(5\,a^7+7\,a^5b^2)\tan(dx+c)^2}{a^8b^3\tan(dx+c)^8+3\,a^9b^2\tan(dx+c)^7+3\,a^{10}b\tan(dx+c)^6+a^{11}\tan(dx+c)^5}+\frac{60\,(a^4b+10\,a^2b^3+14\,b^5)\log(b\tan(dx+c)+a)}{a^9}-\frac{60\,(a^4b+10\,a^2b^3+14\,b^5)\log(\tan(dx+c))}{a^9}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*((6*a^6*b*tan(d*x + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(d*x +
c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(d*x + c)^6 - 110*(
a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(d*x + c)^5 - 15*(a^7 + 10*a^5*b^2 + 14
*a^3*b^4)*tan(d*x + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(d*x + c)^3 - 2*(5*a^
7 + 7*a^5*b^2)*tan(d*x + c)^2)/(a^8*b^3*tan(d*x + c)^8 + 3*a^9*b^2*tan(d*x
+ c)^7 + 3*a^10*b*tan(d*x + c)^6 + a^11*tan(d*x + c)^5) + 60*(a^4*b + 10*a^
2*b^3 + 14*b^5)*log(b*tan(d*x + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b
^5)*log(tan(d*x + c))/a^9)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(294) = 588$.
time = 0.48, size = 1536, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 + 2
9*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(d*x + c)^8 + 2*(10*a^10 + 423*a^
8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(d*x + c)^6 - 15*(a^10
 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(d*x + c)^4 + 20
*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(d*x + c)^2 + 30*(
a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*
a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 +
172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*
b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^
6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7
*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^
7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a
^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3
+ 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(a^6*b^
4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^
6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^
4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 +
50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^6*b^4
+ 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7*b^3 -
 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^7*b^3
- 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a^7*b^3
 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 11*a
^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - 2*(2*(6*a^9*b + 259*a^7*b^3 + 783*a^5*b^5 + 340*a^3*b^7 -
 210*a*b^9)*cos(d*x + c)^7 - (15*a^9*b + 1141*a^7*b^3 + 3546*a^5*b^5 + 1270
*a^3*b^7 - 1260*a*b^9)*cos(d*x + c)^5 + 5*(151*a^7*b^3 + 483*a^5*b^5 + 100*
a^3*b^7 - 252*a*b^9)*cos(d*x + c)^3 - 15*(9*a^7*b^3 + 29*a^5*b^5 - 6*a^3*b^
7 - 28*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 2*a^11*b^3 - a^9*b^5
)*d*cos(d*x + c)^8 - (9*a^13*b + 5*a^11*b^3 - 4*a^9*b^5)*d*cos(d*x + c)^6 +
 3*(3*a^13*b + a^11*b^3 - 2*a^9*b^5)*d*cos(d*x + c)^4 - (3*a^13*b - a^11*b^
3 - 4*a^9*b^5)*d*cos(d*x + c)^2 - (a^11*b^3 + a^9*b^5)*d - ((a^14 - 2*a^12*
b^2 - 3*a^10*b^4)*d*cos(d*x + c)^7 - (2*a^14 - 7*a^12*b^2 - 9*a^10*b^4)*d*c
os(d*x + c)^5 + (a^14 - 8*a^12*b^2 - 9*a^10*b^4)*d*cos(d*x + c)^3 + 3*(a^12
*b^2 + a^10*b^4)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**4, x)
```

Giac [A]
time = 0.73, size = 428, normalized size = 1.43

$$-\frac{\frac{60\,(a^4b+10\,a^2b^3+14\,b^5)\log(|\tan(dx+c)|)}{a^9}-\frac{60\,(a^4b^2+10\,a^2b^4+14\,b^6)\log(|b\tan(dx+c)+a|)}{a^9b}+\frac{5\,\left(22\,a^4b^4\tan(dx+c)^3+220\,a^2b^6\tan(dx+c)^3+308\,b^8\tan(dx+c)^3+75\,a^5b^3\tan(dx+c)^2+720\,a^3b^5\tan(dx+c)^2+987\,ab^7\tan(dx+c)^2+87\,a^6b^2\tan(dx+c)+792\,a^4b^4\tan(dx+c)+1059\,a^2b^6\tan(dx+c)+35\,a^7b+294\,a^5b^3+381\,a^3b^5\right)}{(b\tan(dx+c)+a)^3a^9}-\frac{137\,a^4b\tan(dx+c)^5+1370\,a^2b^3\tan(dx+c)^5+1918\,b^5\tan(dx+c)^5-15\,a^5\tan(dx+c)^4-300\,a^3b^2\tan(dx+c)^4-525\,ab^4\tan(dx+c)^4+60\,a^4b\tan(dx+c)^3+150\,a^2b^3\tan(dx+c)^3-10\,a^5\tan(dx+c)^2-50\,a^3b^2\tan(dx+c)^2+15\,a^4b\tan(dx+c)-3\,a^5}{a^9\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(abs(tan(d*x + c)))/a^9 - 60*(a^
4*b^2 + 10*a^2*b^4 + 14*b^6)*log(abs(b*tan(d*x + c) + a))/(a^9*b) + 5*(22*a
^4*b^4*tan(d*x + c)^3 + 220*a^2*b^6*tan(d*x + c)^3 + 308*b^8*tan(d*x + c)^3
 + 75*a^5*b^3*tan(d*x + c)^2 + 720*a^3*b^5*tan(d*x + c)^2 + 987*a*b^7*tan(d
*x + c)^2 + 87*a^6*b^2*tan(d*x + c) + 792*a^4*b^4*tan(d*x + c) + 1059*a^2*b
^6*tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*tan(d*x + c) +
a)^3*a^9) - (137*a^4*b*tan(d*x + c)^5 + 1370*a^2*b^3*tan(d*x + c)^5 + 1918*
b^5*tan(d*x + c)^5 - 15*a^5*tan(d*x + c)^4 - 300*a^3*b^2*tan(d*x + c)^4 - 5
25*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(d*x + c)^3 + 150*a^2*b^3*tan(d*x + c
)^3 - 10*a^5*tan(d*x + c)^2 - 50*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x
+ c) - 3*a^5)/(a^9*tan(d*x + c)^5))/d
```

Mupad [B]
time = 5.65, size = 337, normalized size = 1.12

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a+2\,b\tan(c+d\,x))\,(a^4+10\,a^2\,b^2+14\,b^4)}{a\,(4\,a^4\,b+40\,a^2\,b^3+56\,b^5)}\right)\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^9\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^5}+\frac{2\tan(c+d\,x)^2\,(5\,a^2+7\,b^2)}{15\,a^3}-\frac{2\,b\tan(c+d\,x)}{5\,a^2}+\frac{22\,b\tan(c+d\,x)^5\,(a^4+10\,a^2\,b^2+14\,b^4)}{3\,a^6}+\frac{10\,b^2\tan(c+d\,x)^6\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^7}+\frac{4\,b^3\tan(c+d\,x)^7\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^8}-\frac{2\,b\tan(c+d\,x)^3\,(5\,a^2+7\,b^2)}{5\,a^4}}{d\,\left(a^3\tan(c+d\,x)^5+3\,a^2\,b\tan(c+d\,x)^6+3\,a\,b^2\tan(c+d\,x)^7+b^3\tan(c+d\,x)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a + 2*b*tan(c + d*x))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a*(4*a
^4*b + 56*b^5 + 40*a^2*b^3)))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a^9*d) - (1/(5*
a) + (tan(c + d*x)^4*(a^4 + 14*b^4 + 10*a^2*b^2))/a^5 + (2*tan(c + d*x)^2*(
5*a^2 + 7*b^2))/(15*a^3) - (2*b*tan(c + d*x))/(5*a^2) + (22*b*tan(c + d*x)^
5*(a^4 + 14*b^4 + 10*a^2*b^2))/(3*a^6) + (10*b^2*tan(c + d*x)^6*(a^4 + 14*b
^4 + 10*a^2*b^2))/a^7 + (4*b^3*tan(c + d*x)^7*(a^4 + 14*b^4 + 10*a^2*b^2))/
a^8 - (2*b*tan(c + d*x)^3*(5*a^2 + 7*b^2))/(5*a^4))/(d*(a^3*tan(c + d*x)^5
+ b^3*tan(c + d*x)^8 + 3*a^2*b*tan(c + d*x)^6 + 3*a*b^2*tan(c + d*x)^7))
```

3.78 $\int \frac{\csc(x)}{1+\tan(x)}\,dx$

Optimal. Leaf size=26

$$-\tanh^{-1}(\cos(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -arctanh(cos(x))+1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3599, 3189, 3855, 3153, 212}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
```

```
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{1+\tan(x)}\,dx &= \int \frac{\cot(x)}{\cos(x)+\sin(x)}\,dx\\ &= \int \left(\csc(x)+\frac{1}{-\cos(x)-\sin(x)}\right)dx\\ &= \int \csc(x)\,dx+\int \frac{1}{-\cos(x)-\sin(x)}\,dx\\ &= -\tanh^{-1}(\cos(x))-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\cos(x)+\sin(x)\right)\\ &= -\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\frac{-\cos(x)+\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 41, normalized size = 1.58

$$(1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] (1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[S
in[x/2]]
```

Maple [A]
time = 0.13, size = 26, normalized size = 1.00

method	result	size
default	$-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	26

risch	$-\frac{\sqrt{2}\,\ln\left(e^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\ln\left(e^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}+\ln\left(e^{ix}-1\right)-\ln\left(e^{ix}+1\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(1+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))+ln(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.
time = 0.53, size = 50, normalized size = 1.92

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(cos
(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.
time = 0.37, size = 55, normalized size = 2.12

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)-3}{2\cos(x)\sin(x)+1}\right)-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*cos
(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{\tan(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + 1), x)
```

Giac [A]
time = 0.44, size = 44, normalized size = 1.69

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1/
2*x) - 2)) + log(abs(tan(1/2*x)))
```

Mupad [B]
time = 3.88, size = 38, normalized size = 1.46

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\sqrt{2}\,\mathrm{atanh}\left(\frac{5\sqrt{2}\tan\left(\frac{x}{2}\right)+2\sqrt{2}}{7\tan\left(\frac{x}{2}\right)+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1)),x)
```

```
[Out] log(tan(x/2)) - 2^(1/2)*atanh((5*2^(1/2)*tan(x/2) + 2*2^(1/2))/(7*tan(x/2)
+ 3))
```

3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$\frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)\sin^{2+m}(c-}{d(2+m)}$$

```
[Out] 3*a^2*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(
2+m)+b^3*hypergeom([2, 2+1/2*m],[3+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(4+m)/d/
(4+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*s
in(d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*b^2*hypergeom([3/2, 3/2+1/
2*m],[5/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(
1/2)/d/(3+m)
```

Rubi [A]
time = 0.33, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4486, 2722, 2644, 371, 2657}

$$\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{3ab^2\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{m+3}(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(c+dx)\right)}{d(m+3)}+\frac{b^3\sin^{m+4}(c+dx)\,{}_2F_1\left(2,\frac{m+4}{2};\frac{m+6}{2};\sin^2(c+dx)\right)}{d(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))
/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)
/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m
)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(4 + m))/(d*(4 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^m(c+dx)+3a^2b\sec(c+dx)\sin^{1+m}(c+dx)+3ab^2\sec^2\right.$$

$$= a^3\int \sin^m(c+dx)\,dx+\left(3a^2b\right)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx+$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3}{}$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3}{}$$

Mathematica [A]
time = 2.82, size = 205, normalized size = 0.90

$$\frac{\sin^{1+m}(c+dx)\left(\frac{a^3\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)}{1+m}+b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)}{2+m}+b\left(\frac{b\,{}_2F_1\left(2,\frac{4+m}{2};\frac{6+m}{2};\sin^2(c+dx)\right)\sin^2(c+dx)}{4+m}+\frac{3a\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(c+dx)\right)\tan(c+dx)}{3+m}\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]*(
```

```
(3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 + m)
+ b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/
2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m)))))/d
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \left(\sin^m\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c
) + a^3)*sin(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^3\sin^m\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^m\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3, x)
```

3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=179

$$\frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{2ab\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)\sin^{2+m}(c+}{d(2+m)}$$

```
[Out] 2*a*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+
m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(
d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeom([3/2, 3/2+1/2*m],[5
/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(1/2)/d/
(3+m)
```

Rubi [A]

time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4486, 2722, 2644, 371, 2657}

$$\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{b^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+3}(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(
d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \left(a^2\sin^m(c+dx)+2ab\sec(c+dx)\sin^{1+m}(c+dx)+b^2\sec^2(c+\right.\\&= a^2\int \sin^m(c+dx)\,dx+(2ab)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx+\\&= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\\&= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\end{aligned}$$

Mathematica [A]
time = 1.34, size = 166, normalized size = 0.93

$$\frac{\sin^{1+m}(c+dx)\left(\frac{a^2\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)}{1+m}+\frac{b\sin(c+dx)\left(2a(3+m)\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)+b(2+m)\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(c+dx)\right)\tan(c+dx)\right)}{(2+m)(3+m)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x]*
(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2] + b
*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2,
Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m))))/d
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int (\sin^m (dx + c)) (a + b \tan (dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \sin^m (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^m\,{\left(a+b\tan(c+d\,x)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2, x)
```

3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=109

$$\frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}$$

```
[Out] b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+m)+a
*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)
^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4486, 2722, 2644, 371}

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
 && !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx &= \int \left(a\sin^m(c+dx)+b\sec(c+dx)\sin^{1+m}(c+dx)\right)\,dx\\ &= a\int \sin^m(c+dx)\,dx+b\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\mathrm{Su}}{}\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\,{}_2F}{}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 109, normalized size = 1.00

$$\frac{a\sqrt{\cos^2(c+dx)}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)\sin^{1+m}(c+dx)}{d(1+m)}+\frac{b\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
+ d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\sin^m(dx+c)\right)\left(a+b\tan(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(c + dx)) \sin^m(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^m\,\left(a+b\,\tan(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x)),x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x)), x)
```

3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=765

$$\frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m}{ad(1+m)}$$

```
[Out] 2^(1+m)*hypergeom([1+m, 1/2+1/2*m],[3/2+1/2*m],-tan(1/2*d*x+1/2*c)^2)*tan(1
/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*
x+1/2*c)^2)^m/a/d/(1+m)+2^(1+m)*b*AppellF1(1+1/2*m,1,1+m,2+1/2*m,a^2*tan(1/
2*d*x+1/2*c)^2/(b-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2
*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)
^2)^m/d/(2+m)/(b-(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)-2^(1+m)*b*AppellF1(1+1/2*
m,1,1+m,2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x
+1/2*c)^2)*tan(1/2*d*x+1/2*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2
))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(2+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))
+2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b
-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*
x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(b-
(a^2+b^2)^(1/2))^2/(a^2+b^2)^(1/2)-2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2
+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2
)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+t
an(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))^2
```

Rubi [A]
time = 3.47, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 6851, 6860, 371, 973, 524}

$$\frac{b2^{m+1}\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+2}{2};m+1,1;\frac{m+4}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)}{d(m+2)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)}-\frac{b2^{m+1}\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+2}{2};m+1,1;\frac{m+4}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)}{d(m+2)\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}+b\right)}+\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+3}{2};m+1,1;\frac{m+5}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)}{d(m+3)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2}-\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+3}{2};m+1,1;\frac{m+5}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)}{d(m+3)\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}+b\right)^2}+\frac{2^{m+1}\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m {}_2F_1\left(\frac{m+1}{2},m+1;\frac{m+3}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(a*d*(1 + m)) + (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m,
 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^2
+ b^2])^2]*Tan[(c + d*x)/2]^2*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m
*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^2])*d*(2 +
m)) - (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^2*
```

```
(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(
Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*d*(2 + m)) + (2^(1 + m)*a*b*AppellF1[
(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^
2)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(
c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^
2 + b^2])^2*d*(3 + m)) - (2^(1 + m)*a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2
]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\text{Subst}\left(\int \frac{2^m(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2^{1+m}\text{Subst}\left(\int \frac{(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\Big(\int}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\Big(\int}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\Big(\int}{ad}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)}{ad(1+m)}$$

Mathematica [F]

time = 15.05, size = 0, normalized size = 0.00

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
[Out] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\sin^m\left(dx+c\right)}{a+b\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
[Out] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^m\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^m}{a+b\,\tan\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m/(a + b*tan(c + d*x)),x)
```

```
[Out] int(sin(c + d*x)^m/(a + b*tan(c + d*x)), x)
```

3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}(\sin^m(c+dx)(a+b\tan(c+dx))^n,x)$$

```
[Out] CannotIntegrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Rubi [A]
time = 1.82, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A]
time = 3.56, size = 0, normalized size = 0.00

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\sin^m\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^n \sin^m (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin (c + d\,x)^m \, (a + b \tan(c + d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n, x)
```

3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=435

$$-\frac{\left(ab^2n(5a^2+b^2(3+2n))+\sqrt{-b^2}\left(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2)\right)\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b}{a-}\right.}{16b\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$

```
[Out] -1/16*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n*
(5*a^2+b^2*(3+2*n))-(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/
2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))-1/16*hype
rgeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*b^2*n*(5*a^2+b^2
*(3+2*n))+(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/2))*(a+b*t
an(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/4*cos(d*x+c)^4*(b
+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d-1/8*cos(d*x+c)^2*(a+b*tan
(d*x+c))^(1+n)*(b*(a^2*(7-n)+b^2*(5+n))+a*(5*a^2+b^2*(3+2*n))*tan(d*x+c))/(
a^2+b^2)^2/d
```

Rubi [A]

time = 0.62, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3597, 1663, 845, 70}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a(5a^2+b^2(2n+3))\tan(c+dx)+b(a^2(7-n)+b^2(n+5))\right)(a+b\tan(c+dx))^{n+1}}{8d\left(a^2+b^2\right)^2}-\frac{\left(ab^2n(5a^2+b^2(2n+3))+\sqrt{-b^2}\left(3a^4+a^2b^2(-n^2+6n+6)+b^4(n^2+4n+3)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{16bd(n+1)\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n(5a^2+b^2(2n+3))-\sqrt{-b^2}\left(3a^4+a^2b^2(-n^2+6n+6)+b^4(n^2+4n+3)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{16bd(n+1)\left(a^2+b^2\right)^2\left(a+\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/16*((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 +
6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a +
b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b*(a^2 + b
^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqr
t[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[
c + d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c
 + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)
*d) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5
 + n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^n}{(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+}{}$$

$$= -\frac{\Big(ab^2n(5a^2+b^2(3+2n)) + \sqrt{-b^2}\,(3a^4+a^2b^2(6+6n-n^2)+b^4}{16b\left(a^2+b^2\right)^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 910 vs. 2(435) = 870.

time = 6.63, size = 910, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n)) -
 (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)) - (C
os[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(b^2*(
a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c
 + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) - a*b^2*n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((a^2*Sqrt[
-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n,
```

```
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(
a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]^2*(a + b*Ta
n[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 - n) + b*(a*(-
3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)) -
(((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n -
 n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt
[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 +
a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/(4*(a^2 + b^2)))
/d
```

Maple [F]
time = 1.39, size = 0, normalized size = 0.00

$$\int \left(\sin^{4}\left(dx+c\right)\right) \left(a+b\tan\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{n}\sin^{4}\left(c+dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^4\,{\left(a+b\,\tan(c+d\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=276

$$-\frac{\left(ab^2n+\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4b\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d(1+n)}-\frac{\left(ab^2n-\sqrt{-b\right.}}{}$$

```
[Out] -1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n-(
a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)/d/(1+n)/(a+
(-b^2)^(1/2))-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)
))*(a*b^2*n+(a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2
)/d/(1+n)/(a-(-b^2)^(1/2))-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c
))^(1+n)/(a^2+b^2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3597, 1663, 845, 70}

$$-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(n+1)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)}-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/4*((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n
, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n
))/(b*(a^2 + b^2)*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n - Sqrt[-b^2]*(a^2
 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
 + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(a + Sqrt[-b
^2])*d*(1 + n)) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])
^(1 + n))/(2*(a^2 + b^2)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
```

```
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^n}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{\text{Subst}(}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{\text{Subst}(}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{(ab^2n}{}$$

$$= -\frac{\left(ab^2n+\sqrt{-b^2}\,(a^2+b^2(1+n))\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+}{a-\sqrt{-b^2}}\right.}{4b(a^2+b^2)\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A]

time = 1.25, size = 270, normalized size = 0.98

$$\frac{\left(\left(a^3\sqrt{-b^2}+a^2b^2(-1+n)-b^4(1+n)-a(-b^2)^{3/2}(1+2n)\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)-\left(a^3\sqrt{-b^2}-a^2b^2(-1+n)+b^4(1+n)-a(-b^2)^{3/2}(1+2n)\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)+2b(a^2+b^2)(1+n)\cos(c+dx)(b\cos(c+dx)+a\sin(c+dx))\right)(a+b\tan(c+dx))^{1+n}}{4b(a^2+b^2)\left(-a+\sqrt{-b^2}\right)\left(a+\sqrt{-b^2}\right)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2*
n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]
)] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(1 +
 2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])] + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c + d*
x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])*(a +
Sqrt[-b^2])*d*(1 + n))
```

Maple [F]
time = 1.14, size = 0, normalized size = 0.00

$$\int \left(\sin^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^n \sin^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin{(c + d\,x)}^2\, (a + b\tan(c + d\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 67}

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.94, size = 48, normalized size = 1.00

$$\frac{b\,_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^n\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2, x)
```

3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=140

$$\frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d}-\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad}+\frac{b(6a^2+b^2(2-3n+n^2)}{}$$

```
[Out] 1/6*b*(2-n)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/3*cot(d*x+c)^3*(a+b
*tan(d*x+c))^(1+n)/a/d+1/6*b*(6*a^2+b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+
n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^4/d/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3597, 964, 79, 67}

$$\frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{6a^2d}+\frac{b(6a^2+b^2(n^2-3n+2))\,(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{6a^4d(n+1)}-\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{n+1}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c
+ d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n +
 n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Ta
n[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 964

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^n\left(b^2+x^2\right)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} - \frac{b\text{Subst}\left(\int \frac{(a+x)^n\left(b^2(2-n)-3ax\right.}{x^3}\right.}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\,\mathrm{t}}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\,\mathrm{t}}{3ad}$$

Mathematica [A]
time = 1.29, size = 78, normalized size = 0.56

$$\frac{b\left(a^2\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)+b^2\,{}_2F_1\left(4,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)\right)(a+b\tan(c+dx))^{1+n}}{a^4d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*Hy
pergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c + d*
x])^(1 + n))/(a^4*d*(1 + n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(\csc^4(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^n \csc^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4, x)
```

3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\sin^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Rubi [A]
time = 1.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A]
time = 3.18, size = 0, normalized size = 0.00

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin\left(c+d\,x\right)^3\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=22

$$\text{Int}(\sin(c+dx)(a+b\tan(c+dx))^n,x)$$

```
[Out] CannotIntegrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [A]
time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A]
time = 2.42, size = 0, normalized size = 0.00

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin(dx+c)\,(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^n \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sin(c + d\,x)\ (a + b\tan(c + d\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}(\csc(c+dx)(a+b\tan(c+dx))^n, x)$$

```
[Out] CannotIntegrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [A]
time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A]
time = 1.70, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^n \csc(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b\tan(c + d\,x))^n}{\sin(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x), x)
```

3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\csc^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Rubi [A]
time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A]
time = 14.83, size = 0, normalized size = 0.00

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\csc^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^n \csc^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```